1 Matrix Elements and Completeness Relations

Writing an operator in matrix notation in its own basis is easy: it is diagonal with the eigenvalues on the diagonal.

What if I want to calculate the matrix elements using a different basis??

The eigenvalue equation tells me what happens when an operator acts on its own eigenstate. For example: $\hat{S}_y \left|\pm\right\rangle_y = \pm\frac{\hbar}{2}\left|\pm\right\rangle_y$

In Dirac bra-ket notation, to know what an operator does to a ket, I need to write the ket in the basis that is the eigenstates of the operator (in order to use the eigenvalue equation.)

One way to do this is to stick completeness relationships into the braket:

$$\left\langle+\right|\hat{S}_y\left|+\right\rangle = \left\langle+\right|(I)\hat{S}_y(I)\left|+\right\rangle$$

where I is the identity operator: $I = \left|+\right\rangle_{yy}\left\langle+\right| + \left|-\right\rangle_{yy}\left\langle-\right|$. This effectively rewrites the $\left|+\right\rangle$ in the $\left|\pm\right\rangle_y$ basis.

Find the top row matrix elements of the operator $\hat{S}_y$ in the S_z basis by inserting completeness relations into the brakets. (The answer is already on the Spins Reference Sheet, but I want you to demonstrate the calculation.)

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