1 Lines in Polar Coordinates

(2, 2, 2 pts)
(Algebra involving trigonometric functions) Purpose: Practice with polar equations.

The general equation for a straight line in polar coordinates is given by:

$$r(\phi) = \frac{r_0}{\cos(\phi - \delta)} \tag{1}$$

where r_0 and δ are constant parameters. Find the polar equation for the straight lines below. You do NOT need to evaluate any complicated trig or inverse trig functions. You may want to try plotting the general polar equation to figure out the roles of the parameters.

(a) $y = 3$

(b) $x = 3$

(c) $y = -3x + 2$

2 Find Force Law: Logarithmic Spiral Orbit

(4 pts) (Use the equation for orbit shape.) *Gain experience with unusual force laws.*

In science fiction movies, characters often talk about a spaceship "spiralling in" right before it hits the planet. But all orbits in a $1/r^2$ force are conic sections, not spirals. This spiralling in happens because the spaceship hits atmosphere and the drag from the atmosphere changes the shape of the orbit. But, in an alternate universe, we might have other force laws.

In class, we discussed how to calculate the shape of the orbit for an inverse square potential. More generally, the equation for the orbit of a mass μ under the influence of a central force $f(r)$ is given by:

$$\frac{d^2u}{d\phi^2} + u = -\frac{\mu}{\ell^2}\frac{1}{u^2} f\left(\frac{1}{u}\right) \tag{2}$$

$$\Rightarrow f\left(\frac{1}{u}\right) = -\frac{\ell^2}{\mu}u^2\left(\frac{d^2u}{d\phi^2} + u\right) \tag{3}$$

where $u = r^{-1}$.

Find the force law for a mass μ, under the influence of a central-force field, that moves in a logarithmic spiral orbit given by $r = ke^{\alpha\phi}$, where k and α are constants.

3 Hockey

(2, 2, 2, 2, 2, 2 pts)

(Synthesis Problem: Brings together several different concepts from this unit.) *Use effective potential diagrams for other than $1/r^2$ forces.*

Consider the frictionless motion of a hockey puck of mass m on a perfectly circular bowl-shaped ice rink with radius a. The central region of the bowl ($r < 0.8a$) is perfectly flat and the sides of the ice bowl smoothly rise to a height h at $r = a$.

(a) Sketch the potential energy for this system (just the potential energy, not the effective potential). Set the zero of potential energy at the top of the sides of the bowl.

(b) Situation 1: the puck is initially moving radially outward from the exact center of the rink. What minimum velocity does the puck need to escape the rink?

(c) Situation 2: a stationary puck, at a distance $\frac{a}{2}$ from the center of the rink, is hit in such a way that it's initial velocity $\vec{v}_0$ is perpendicular to its position vector as measured from the center of the rink. What is the total energy of the puck immediately after it is struck?

(d) In situation 2, what is the angular momentum of the puck immediately after it is struck?

(e) Draw a sketch of the effective potential for situation 2.

(f) In situation 2, for what minimum value of $\vec{v}_0$ does the puck just escape the rink?

4 Confidence Rating

(1 pt) After solving each problem on the assignment, indicate your answers to the following questions for each problem. Answer for the problem as a whole, even if the problem has multiple parts.

(a) **Question Confidence** How confident are you that you are interpreting the problem the way the instructor intends?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

For the rest of the questions, assume you have interpreted the problem correctly

(b) **Problem Confidence** How confident are you that you could independently come up with a correct solution process to a similar problem on a future problem set?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

(c) **Answer Confidence** How confident are you that your final answer to this question is correct (not solution process)?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

(d) **Makes Sense** To what degree do you understand how your answer fits (or does not fit) the physical or mathematical situation of the problem?

VN	N	LN	IDK	LF	F	VF
Very confident answer does NOT fit	Somewhat confident answer does NOT fit	Leaning toward the answer does NOT fit	Don't know if answer fits or not	Leaning toward the answer fits	Somewhat confident the answer fits	Very confident answer fits