1 Working with Representations on the Ring

(Moved from HW6 in 2025 Spring 4,2,2,4,4 pts)

The following are 3 different representations for the **same** state on a quantum ring for $r_0 = 1$

$$|\Phi_a\rangle = i\sqrt{\frac{2}{12}}\,|3\rangle - \sqrt{\frac{1}{12}}\,|1\rangle + \sqrt{\frac{3}{12}}e^{i\frac{\pi}{4}}\,|0\rangle - i\sqrt{\frac{2}{12}}\,|{-1}\rangle + \sqrt{\frac{4}{12}}\,|{-3}\rangle \tag{1}$$

$$|\Phi_b\rangle \doteq \begin{pmatrix} \vdots \\ i\sqrt{\frac{2}{12}} \\ 0 \\ -\sqrt{\frac{1}{12}} \\ \sqrt{\frac{3}{12}}e^{i\frac{\pi}{4}} \\ -i\sqrt{\frac{2}{12}} \\ 0 \\ \sqrt{\frac{4}{12}} \\ \vdots \end{pmatrix} \begin{matrix} \\ \\ \\ \\ \leftarrow m = 0 \\ \\ \\ \\ \\ \end{matrix} \tag{2}$$

$$\Phi_c(\phi) \doteq \sqrt{\frac{1}{24\pi}}\left(i\sqrt{2}e^{i3\phi} - e^{i\phi} + \sqrt{3}e^{i\frac{\pi}{4}} - i\sqrt{2}e^{-i\phi} + \sqrt{4}e^{-i3\phi}\right) \tag{3}$$

(a) With each representation of the state given above, explicitly calculate the probability that $L_z = -1\hbar$. Then, calculate all other non-zero probabilities for values of L_z with a method/representation of your choice.

(b) Explain how you could be sure you calculated all of the non-zero probabilities.

(c) If you measured the z-component of angular momentum to be $3\hbar$, what would the state of the particle be immediately after the measurement is made?

(d) With each representation of the state given above, explicitly calculate the probability that $E = \frac{9}{2}\frac{\hbar^2}{I}$. Then, calculate all other non-zero probabilities for values of E with a method of your choice.

(e) If you measured the energy of the state to be $\frac{9}{2}\frac{\hbar^2}{I}$, what would the state of the particle be immediately after the measurement is made?

2 Ring Function

(2,2,2,2,2 pts)

Consider the normalized wavefunction $\Phi(\phi)$ for a quantum mechanical particle of mass μ constrained to move on a circle of radius r_0, given by:

$$\Phi(\phi) = \frac{N}{2 + \cos(3\phi)} \tag{4}$$

where N is the normalization constant.

(a) Find N.

(b) Plot this wave function.

(c) Plot the probability density.

(d) Find the probability that if you measured L_z you would get $3\hbar$.

(e) What is the expectation value of L_z in this state?

3 Ring Table

(12 pts)

Attached, you will find a table showing different representations of physical quantities associated with a quantum particle confined to a ring. Fill in all of the missing entries. Hint: You may look ahead. We filled out a number of the entries throughout the table to give you hints about what the forms of the other entries might be. pdf link for the Table or doc link for the Table

4 Confidence Rating

(1 pt) After solving each problem on the assignment, indicate your answers to the following questions for each problem. Answer for the problem as a whole, even if the problem has multiple parts.

(a) **Question Confidence** How confident are you that you are interpreting the problem the way the instructor intends?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

For the rest of the questions, assume you have interpreted the problem correctly

(b) **Problem Confidence** How confident are you that you could independently come up with a correct solution process to a similar problem on a future problem set?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

(c) **Answer Confidence** How confident are you that your final answer to this question is correct (not solution process)?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

(d) **Makes Sense** To what degree do you understand how your answer fits (or does not fit) the physical or mathematical situation of the problem?

VN	N	LN	IDK	LF	F	VF
Very confident answer does NOT fit	Somewhat confident answer does NOT fit	Leaning toward the answer does NOT fit	Don't know if answer fits or not	Leaning toward the answer fits	Somewhat confident the answer fits	Very confident answer fits