1 Recurrence Relations

In 2025, do part (a) only. (4 pts)

For each of the problems below, suppose you have been solving a differential equation using power series methods around the indicated point and you have derived the indicated recurrence relation. Write out the first five nonzero terms in the power series expansion. If the recurrence relation allows two solutions, write out the first four nonzero terms in each such solution.

(a) In an expansion around the point $z = 1$, the recurrence relation is:

$$a_{n+1} = \frac{1}{n+1}\, a_n$$

(b) In an expansion around the point $z = 0$, the recurrence relation is:

$$a_{n+2} = -\frac{(5-n)(6+n)}{(n+2)(n+1)}\, a_n$$

2 ODE Power Series Solutions One

(8 pts)

Consider the differential equation $y'' - 2y' + y = 0$.

(a) Use the power series method to find the first six terms in each of two independent solutions to this differential equation.

(b) Solve this differential equation using a different method and show that your answers are the same as part a.

3 Quantum Particle in a 2D Box with Time Dependence

(16 pts) The eigenstates for a quantum mechanical particle inside a 2-dimensional infinite potential well with sides of length L_x, L_y are

$$|n, m\rangle \doteq \sqrt{\frac{2}{L_x}} \sin\frac{n_x \pi x}{L_x} \sqrt{\frac{2}{L_y}} \sin\frac{n_y \pi y}{L_y}$$

(a) Find an exact expression for the initial wave function given by:

$$\psi(x, y, 0) = \frac{30}{\sqrt{L_x^5 L_y^5}} (L_x x - x^2)(L_y y - y^2)$$

I have chosen coordinates so that one of the corners of the box is at the origin and all of the box is in the first quadrant (i.e. all positive values of the spatial coordinates).

(b) Find an exact expression for the wave function as a function of time.

(c) Bonus points: Plot an approximation for the probability density at $t = 0$ and at an interesting later time. Explain why you chose the later time that you did. Explain how you chose your approximation scheme and why.

4 Confidence Rating

(1 point) After solving each problem on the assignment, indicate your answers to the following questions for each problem. Answer for the problem as a whole, even if the problem has multiple parts.

(a) **Question Confidence** How confident are you that you are interpreting the problem the way the instructor intends?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

For the rest of the questions, assume you have interpreted the problem correctly

(b) **Problem Confidence** How confident are you that you could independently come up with a correct solution process to a similar problem on a future problem set?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

(c) **Answer Confidence** How confident are you that your final answer to this question is correct (not solution process)?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

(d) **Makes Sense** To what degree do you understand how your answer fits (or does not fit) the physical or mathematical situation of the problem?

VN	N	LN	IDK	LF	F	VF
Very confident answer does NOT fit	Somewhat confident answer does NOT fit	Leaning toward the answer does NOT fit	Don't know if answer fits or not	Leaning toward the answer fits	Somewhat confident the answer fits	Very confident answer fits