# 1 Eigenvalues for Different Systems

(a) Fill in the following table with the appropriate eigenvalues for each operator for each system.

| | $\lvert m\rangle$ particle on a ring | $\lvert \ell, m\rangle$ particle on a sphere | $\lvert n, \ell, m\rangle$ Hydrogen atom |
|---|---|---|---|
| $L_z$ | | | |
| $L^2$ | | | |
| $H$ | | | |

(b) Write the Hamiltonian for each of the following systems explicitly in the position representation (i.e., differential operators).

| | $\lvert m\rangle$ particle on a ring | $\lvert \ell, m\rangle$ particle on a sphere | $\lvert n, \ell, m\rangle$ Hydrogen atom |
|---|---|---|---|
| $H$ | | | |

# 2 SP not Hybrid

A hydrogen atom is initially in the state $|\Psi(t=0)\rangle = \frac{1}{\sqrt{2}}\left(|1,0,0\rangle + |2,1,0\rangle\right)$.

(a) If you measure the energy of this state, what possible values could you obtain?

(b) What is $|\Psi(t)\rangle$, where $t > 0$?

(c) Calculate the expectation value $\langle \hat{L}^2 \rangle$ in this state, as a function of time. Did you expect this answer? Please explain your reasoning.

(d) Write $|\Psi(t)\rangle$ in wave function notation.

# 3 Quantum Cylinder

Consider a quantum particle confined to the surface of a cylinder (not including the endcaps). Let the height of the cylinder be equal to half its circumference.

(a) Write down the Hamiltonian for this system, the Schrodinger equation, and any relevant boundary conditions.

(b) Determine the energy eigenfunctions and energy eigenvalues of this system. (You may find it valuable to base your answer on systems you have previously studied!)

(c) Explicitly write out the energy, the state in ket notation, and the full wave function (including time dependence) for the ground state and the first excited state.