MULTIVARIABLE CALCULUS PICTIONARY

Recorder: ________________________

Task Master: ________________________ Cynic: ________________________

Working in groups of 3, solve as many of the problems below as possible. Try to resolve questions within the group before asking for help. Each person should turn in their solutions to two surfaces. Show your work! Full credit will only be given if your answer is supported by calculations and/or explanations as appropriate.

Consider one of the eight surfaces given below.

A $z - \frac{x^2}{4} - \frac{y^2}{4} = 0$	**B** $18x^2 - \frac{9}{2}y^2 = 18 + 2z^2$	**C** $z - \frac{x^2}{4} + \frac{y^2}{4} = 0$	**D** $\frac{x^2}{9} + \frac{y^2}{4} = 1 - \frac{z^2}{16}$
E $\frac{x^2}{9} + \frac{y^2}{4} = \frac{z^2}{16}$	**F** $\frac{4x^2}{9} + y^2 = 4 + \frac{z^2}{4}$	**G** $z - xy = 0$	**H** $9y = x^2$

(1) **Ready** Choose one of the surfaces above, and an appropriate variable x, y, or z – which we'll label as ω.

(2) **Set** For each value of $k = -2, -1, 0, 1, 2$, draw the curve created by intersecting the surface with the plane $\omega = k$, if it exists. You should have (up to) 5 different curves.

(3) **Go!** "Stack" the planes into position on the ω axis. Can you draw the surface? If so, do it! If not, what other information do you need to draw the surface?

(4) **Next...** Go back and try this with another surface, chosen from a different row and column.

Date: January 4, 2010.

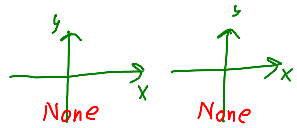
A $z-\frac{x^2}{4}-\frac{y^2}{4}=0$	B $18x^2-\frac{9}{2}y^2=18+2z^2$	C $z-\frac{x^2}{4}+\frac{y^2}{4}=0$	D $\frac{x^2}{9}+\frac{y^2}{4}=1-\frac{z^2}{16}$
E $\frac{x^2}{9}+\frac{y^2}{4}=\frac{z^2}{16}$	F $\frac{4x^2}{9}+y^2=4+\frac{z^2}{4}$	G $z-xy=0$	H $9y=x^2$

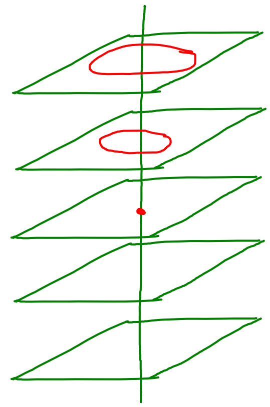
A) Choose z for w, as the rest looks like circles.

$$z=\frac{x^2}{4}+\frac{y^2}{4}$$

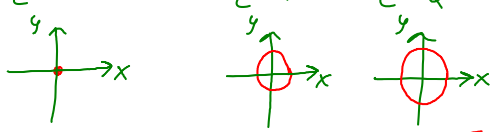
$$z=\frac{r^2}{4}$$

$z=-2$ $\quad$ $z=-1$





$z=0$ $\quad$ $z=1$ $\quad$ $z=2$



$r=0$ $\quad$ $r=2$ $\quad$ $r=2\sqrt{2}$

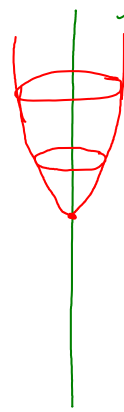
Need way to "connect" layers $\Rightarrow$ use $w=y$, and

view trace when $y=0$: $z=\frac{x^2}{4}+\frac{0}{4}$

which means layers connected by a parabola.

Surface:

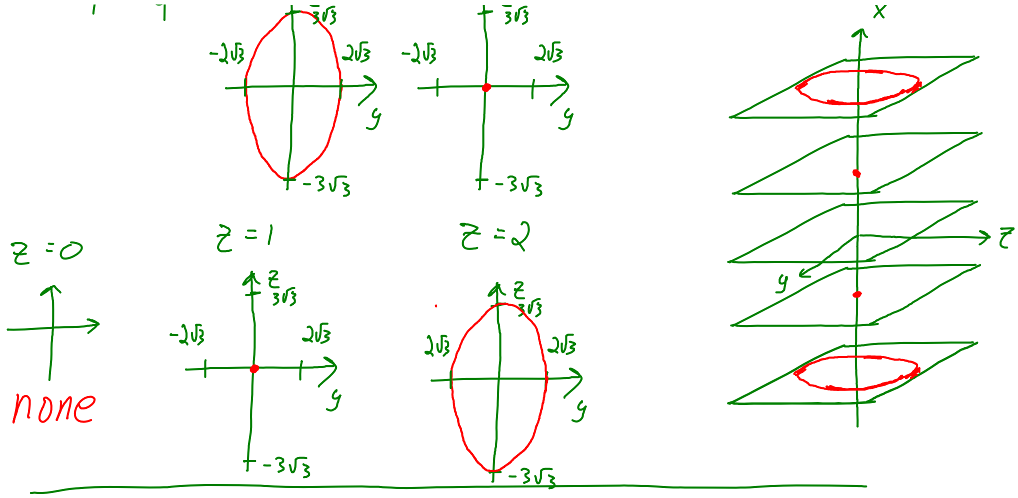
$\boxed{\text{Paraboloid}}$



A $z-\frac{x^2}{4}-\frac{y^2}{4}=0$	B $18x^2-\frac{9}{2}y^2=18+2z^2$	C $z-\frac{x^2}{4}+\frac{y^2}{4}=0$	D $\frac{x^2}{9}+\frac{y^2}{4}=1-\frac{z^2}{16}$
E $\frac{x^2}{9}+\frac{y^2}{4}=\frac{z^2}{16}$	F $\frac{4x^2}{9}+y^2=4+\frac{z^2}{4}$	G $z-xy=0$	H $9y=x^2$

B) Choose x for w, as the rest looks like ellipses.

$x^2-1=\frac{y^2}{4}+\frac{z^2}{9}$ $\quad$ $x=-2$ $\quad$ $z=-1$

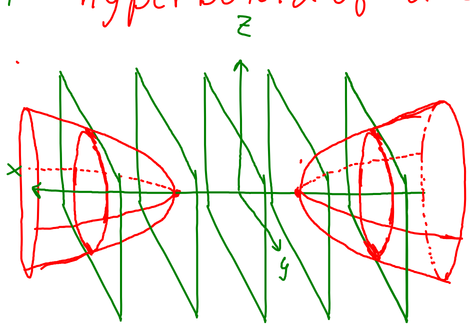


Need way to "connect" layers $\Rightarrow$ use $w = y$, and

view trace when $y=0$: $x^2 - \frac{z^2}{9} = 1 + \frac{0^2}{4}$

which means layers connected by a hyperbola.

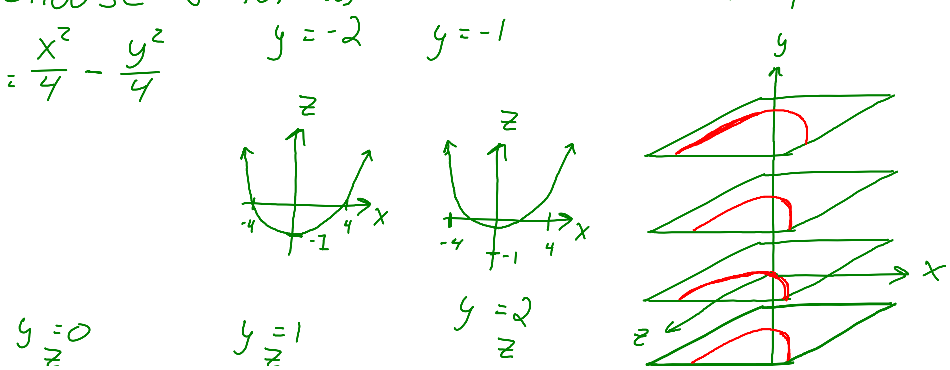
Surface: hyperboloid of 2 sheets.



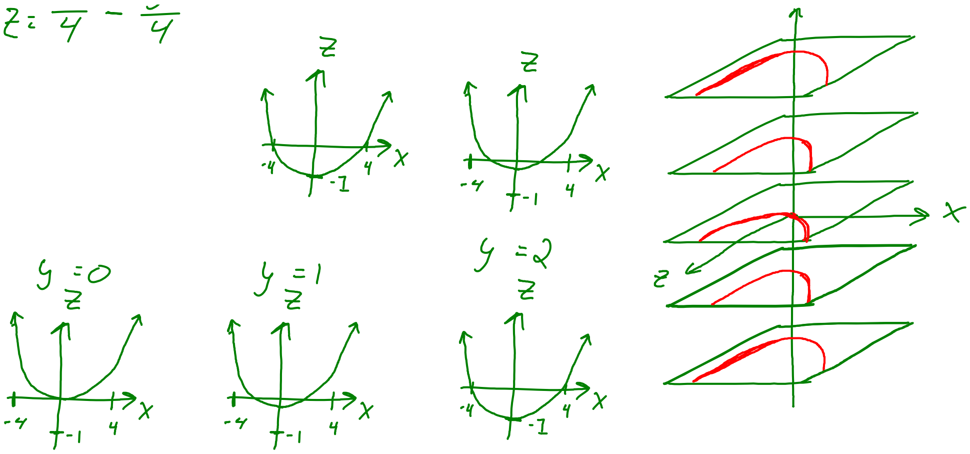
A $z - \frac{x^2}{4} - \frac{y^2}{4} = 0$	**B** $18x^2 - \frac{9}{2}y^2 = 18 + 2z^2$	**C** $z - \frac{x^2}{4} + \frac{y^2}{4} = 0$	**D** $\frac{x^2}{9} + \frac{y^2}{4} = 1 - \frac{z^2}{16}$
E $\frac{x^2}{9} + \frac{y^2}{4} = \frac{z^2}{16}$	**F** $\frac{4x^2}{9} + y^2 = 4 + \frac{z^2}{4}$	**G** $z - xy = 0$	**H** $9y = x^2$

c) Choose y for w, as the rest looks like parabolas.

$z = \frac{x^2}{4} - \frac{y^2}{4}$



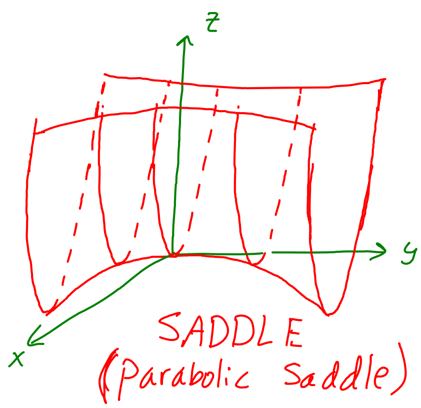
$z = \frac{x^2}{4} - \frac{y^2}{4}$



Need way to "connect" layers $\Rightarrow$ use $w = x$, and view trace when $x = 0$: $z = -\frac{y^2}{4} + \frac{0}{4}$

which means layers connected by a parabola opening downward about the z axis y-z plane.

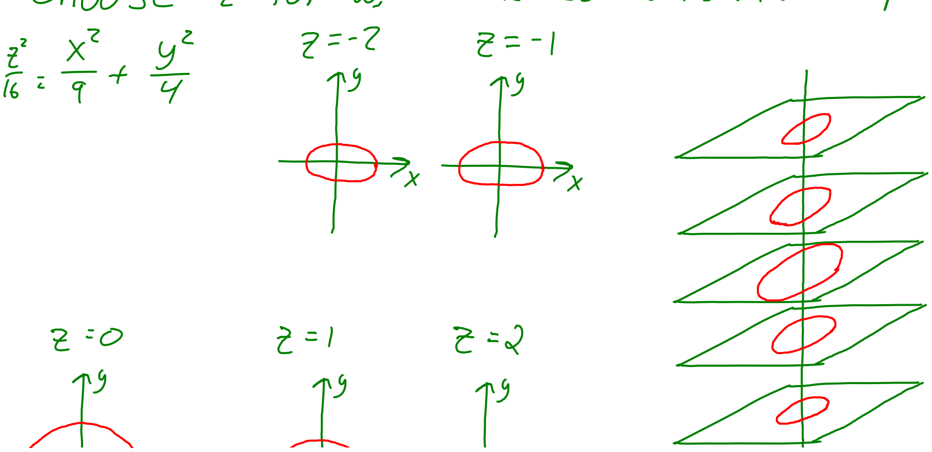
Surface:



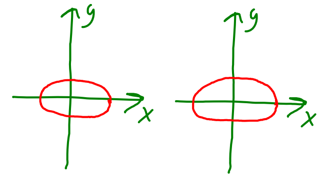
A $z - \frac{x^2}{4} - \frac{y^2}{4} = 0$	**B** $18x^2 - \frac{9}{2}y^2 = 18 + 2z^2$	**C** $z - \frac{x^2}{4} + \frac{y^2}{4} = 0$	**D** $\frac{x^2}{9} + \frac{y^2}{4} = 1 - \frac{z^2}{16}$
E $\frac{x^2}{9} + \frac{y^2}{4} = \frac{z^2}{16}$	**F** $\frac{4x^2}{9} + y^2 = 4 + \frac{z^2}{4}$	**G** $z - xy = 0$	**H** $9y = x^2$

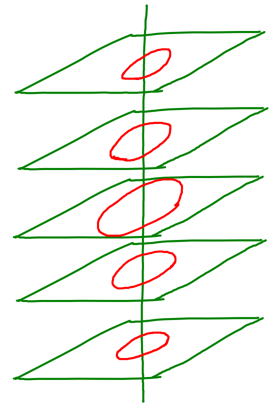
D) Choose z for w, as the rest looks like ellipses.

$1 - \frac{z^2}{16} = \frac{x^2}{9} + \frac{y^2}{4}$

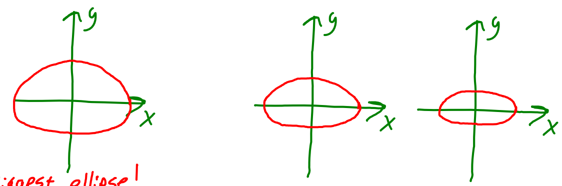


$1-\frac{\_}{16}=\frac{\_}{9}+\frac{\_}{4}$





$z=0$ $\quad z=1$ $\quad z=2$



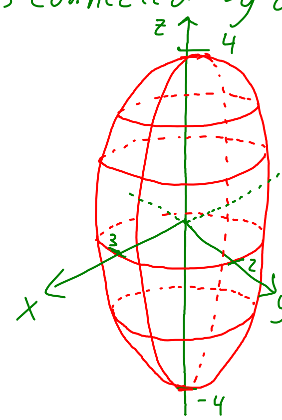
Biggest ellipse!

Need way to "connect" layers $\Rightarrow$ use $w=y$, and

view trace when $y=0$: $1-\frac{y^2}{4}=\frac{x^2}{9}+\frac{z^2}{16}$

which means layers connected by an ellipse.

Surface:

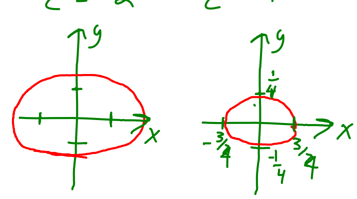


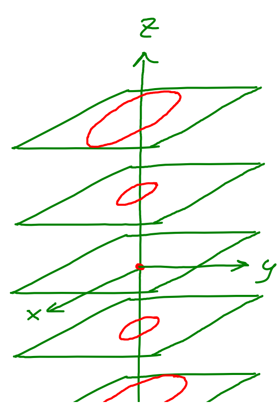
An Ellipse all around... known as an $\boxed{\text{ellipsoid.}}$

A $z-\frac{x^2}{4}-\frac{y^2}{4}=0$	B $18x^2-\frac{9}{2}y^2=18+2z^2$	C $z-\frac{x^2}{4}+\frac{y^2}{4}=0$	D $\frac{x^2}{9}+\frac{y^2}{4}=1-\frac{z^2}{16}$
E $\frac{x^2}{9}+\frac{y^2}{4}=\frac{z^2}{16}$	F $\frac{4x^2}{9}+y^2=4+\frac{z^2}{4}$	G $z-xy=0$	H $9y=x^2$

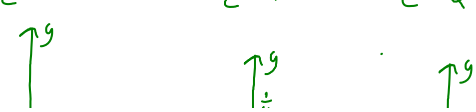
E) Choose z for w, as the rest looks like ellipses.

$\frac{z^2}{16}=\frac{x^2}{9}+\frac{y^2}{4}$ $\quad z=-2$ $\quad z=-1$

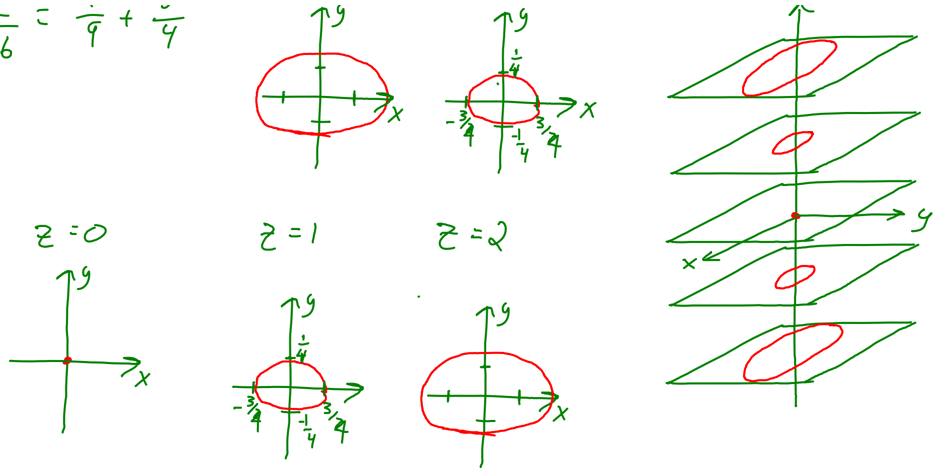




$z=0$ $\quad z=1$ $\quad z=2$



$\frac{z^2}{16} = \frac{x^2}{9} + \frac{y^2}{4}$



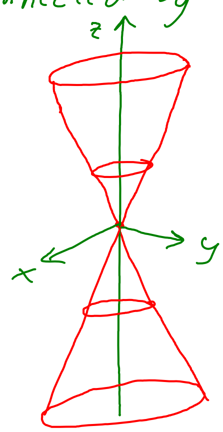
Need way to "connect" layers $\Rightarrow$ use $w = y$, and

view trace when $y = 0$: $9z^2 = 16x^2 \Rightarrow z = \pm\frac{4}{3}x$

which means layers connected by lines.

Surface:

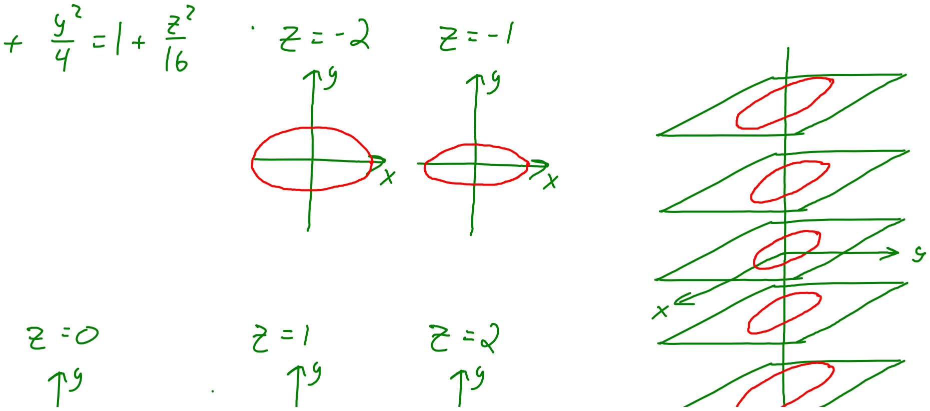
elliptical cone

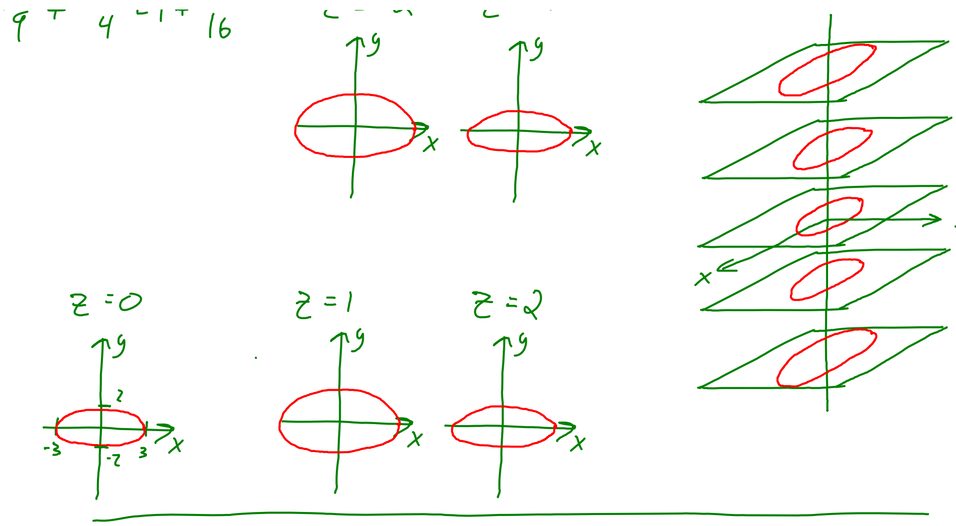


A	$z - \frac{x^2}{4} - \frac{y^2}{4} = 0$	**B**	$18x^2 - \frac{9}{2}y^2 = 18 + 2z^2$	**C**	$z - \frac{x^2}{4} + \frac{y^2}{4} = 0$	**D**	$\frac{x^2}{9} + \frac{y^2}{4} = 1 - \frac{z^2}{16}$
E	$\frac{x^2}{9} + \frac{y^2}{4} = \frac{z^2}{16}$	**F**	$\frac{4x^2}{9} + y^2 = 4 + \frac{z^2}{4}$	**G**	$z - xy = 0$	**H**	$9y = x^2$

F) Choose z for w, as the rest looks like ellipses.

$\frac{x^2}{9} + \frac{y^2}{4} = 1 + \frac{z^2}{16}$





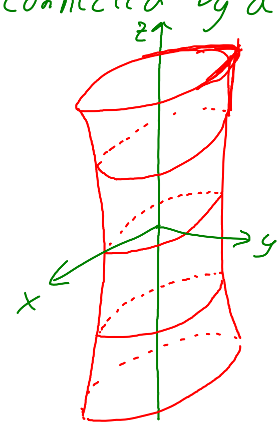
Need way to "connect" layers $\Rightarrow$ use $w = y$, and

view trace when $y = 0$: $\frac{x^2}{9} - \frac{z^2}{16} = 1$

which means layers connected by a hyperbola.

Surface:

hyperboloid of 1 sheet

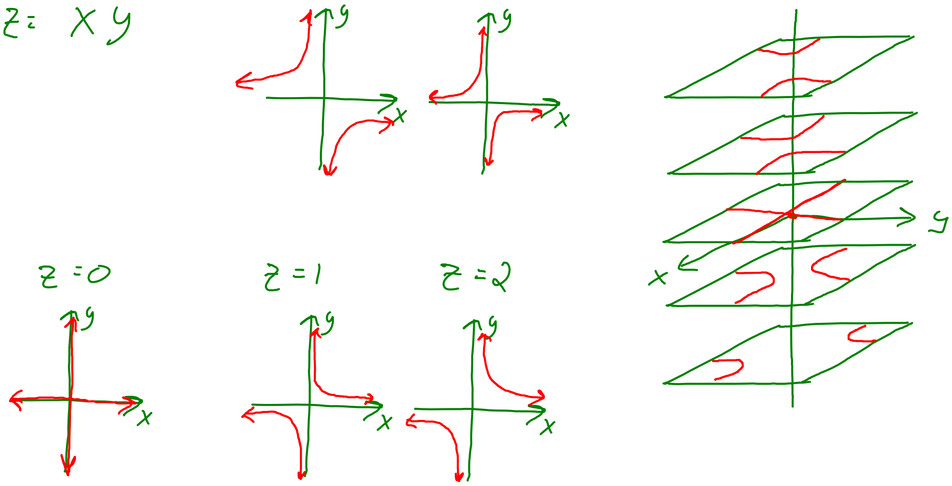


A	$z - \frac{x^2}{4} - \frac{y^2}{4} = 0$	**B**	$18x^2 - \frac{9}{2}y^2 = 18 + 2z^2$	**C**	$z - \frac{x^2}{4} + \frac{y^2}{4} = 0$	**D**	$\frac{x^2}{9} + \frac{y^2}{4} = 1 - \frac{z^2}{16}$
E	$\frac{x^2}{9} + \frac{y^2}{4} = \frac{z^2}{16}$	**F**	$\frac{4x^2}{9} + y^2 = 4 + \frac{z^2}{4}$	**G**	$z - xy = 0$	**H**	$9y = x^2$

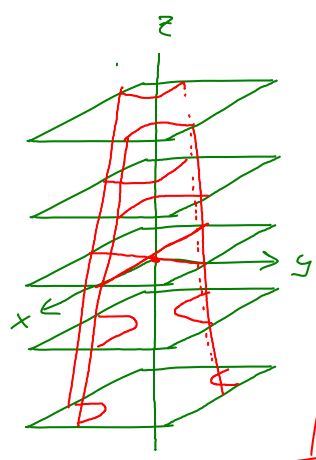
G) Choose z for w, as the rest looks like "$y = \frac{c}{x}$"

$z = -2$ $\quad$ $z = -1$ $\quad$ z

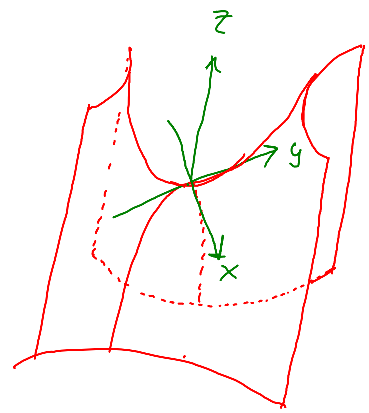
$z = xy$



for $w=y$, we see $z = xw$ is linear for w a constant.



or, rotated a bit:

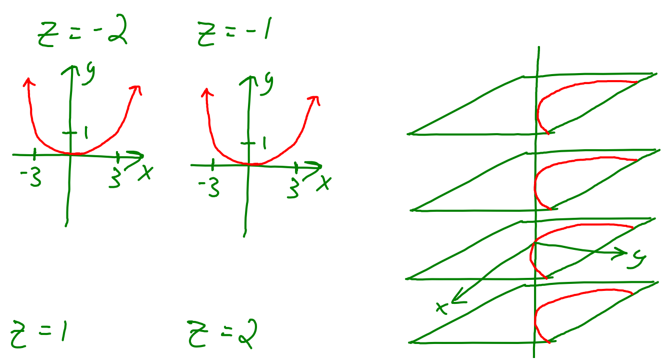


A Saddle

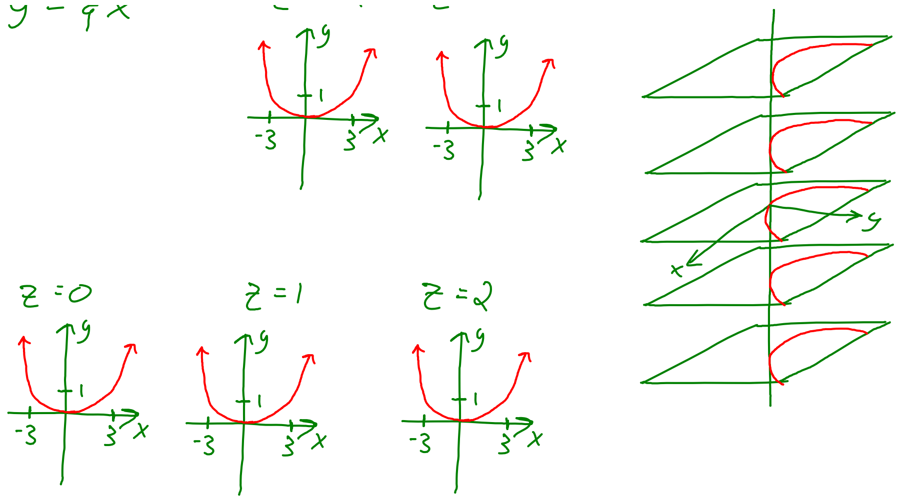
A	$z-\frac{x^2}{4}-\frac{y^2}{4}=0$	B	$18x^2-\frac{9}{2}y^2=18+2z^2$	C	$z-\frac{x^2}{4}+\frac{y^2}{4}=0$	D	$\frac{x^2}{9}+\frac{y^2}{4}=1-\frac{z^2}{16}$
E	$\frac{x^2}{9}+\frac{y^2}{4}=\frac{z^2}{16}$	F	$\frac{4x^2}{9}+y^2=4+\frac{z^2}{4}$	G	$z-xy=0$	H	$9y=x^2$

H) Choose z for w, as the rest looks like parabolas.

$y = \frac{1}{9}x^2$



$z=0$ $z=1$ $z=2$



Need way to "connect" layers, but will be the same parabola for every z,

which means every layer is the same.

Surface:

cylindrical paraboloid