	Ket Representation	Wave Function Representation	Matrix Representation
Operator for z-component of angular momentum	L_z	$-i\hbar\frac{\partial}{\partial\phi}$	$\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \iddots \\ \cdots & 1\hbar & 0 & 0 & \cdots \\ \cdots & 0 & 0\hbar & 0 & \cdots \\ \cdots & 0 & 0 & -1\hbar & \cdots \\ \iddots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenvalues of L_z	$m\hbar$	$m\hbar$	$m\hbar$
Normalized Eigenstates of L_z	$\|m\rangle$	$\Phi_m(\phi)=\sqrt{\frac{1}{2\pi r_0}}e^{im\phi}$	$\ldots,\begin{pmatrix}\vdots\\1\\0\\0\\\vdots\end{pmatrix},\begin{pmatrix}\vdots\\0\\1\\0\\\vdots\end{pmatrix},\begin{pmatrix}\vdots\\0\\0\\1\\\vdots\end{pmatrix}\ \ldots$
Coefficient of m^{th} eigenstates of L_z	$c_m=\langle m\|\Phi\rangle$	$c_m=\int_0^{2\pi}\sqrt{\frac{1}{2\pi r_0}}e^{-im\phi}\,\Phi(\phi)r_0d\phi$	$\begin{pmatrix}\cdots & 1 & \cdots & 0 & \cdots\end{pmatrix}\begin{pmatrix}\vdots\\c_m\\\vdots\\c_0\\\vdots\end{pmatrix}$
Probability of measuring $m\hbar$ for z-component of angular momentum	$P(\hbar m)=\|c_m\|^2=\|\langle m\|\Phi\rangle\|^2$	$P(\hbar m)=\|c_m\|^2=\left\|\int_0^{2\pi}\sqrt{\frac{1}{2\pi r_0}}e^{-im\phi}\,\Phi(\phi)r_0d\phi\right\|^2$	$P(m\hbar)=\|c_m\|^2=\left\|\begin{pmatrix}\cdots & 1 & \cdots & 0 & \cdots\end{pmatrix}\begin{pmatrix}\vdots\\c_m\\\vdots\\c_0\\\vdots\end{pmatrix}\right\|^2$
Expectation value of z-component of angular momentum	$\langle\Phi\|\hat{L}_z\|\Phi\rangle=\sum_m\|c_m\|^2\,m\hbar$	$\langle\Phi\|\hat{L}_z\|\Phi\rangle=\int_0^{2\pi}\Phi^*(\phi)\ \left(-i\hbar\frac{\partial}{\partial\phi}\right)\Phi(\phi)\,r_0d\phi$	$\langle\Phi\|L_z\|\Phi\rangle=\begin{pmatrix}\cdots & c_1^* & c_0^* & c_{-1}^* & \cdots\end{pmatrix}\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \iddots \\ \cdots & 1\hbar & 0 & 0 & \cdots \\ \cdots & 0 & 0\hbar & 0 & \cdots \\ \cdots & 0 & 0 & -1\hbar & \cdots \\ \iddots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}\begin{pmatrix}\vdots\\c_1\\c_0\\c_{-1}\\\vdots\end{pmatrix}$

Particle on a Ring Table 2

	Ket Representation	Wave Function Representation	Matrix Representation
Hamiltonian	$\hat{H}$	$-\frac{\hbar^2}{2I}\frac{d^2}{d\phi^2}$	$\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & ⋰ \\ \cdots & E_1 & 0 & 0 & \cdots \\ \cdots & 0 & E_0 & 0 & \cdots \\ \cdots & 0 & 0 & E_{-1} & \cdots \\ ⋰ & \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & ⋰ \\ \cdots & \hbar^2/2I & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & \hbar^2/2I & \cdots \\ ⋰ & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenvalues of Hamiltonian	$E_m = \frac{\hbar^2}{2I}m^2$	$E_m = \frac{\hbar^2}{2I}m^2$	$E_m = \frac{\hbar^2}{2I}m^2$
Normalized Eigenstates of Hamiltonian	$\lvert m\rangle$	$\Phi_m(\phi) = \sqrt{\frac{1}{2\pi r_0}}e^{im\phi}$	$\ldots, \begin{pmatrix} \vdots \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \begin{pmatrix} \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}, \begin{pmatrix} \vdots \\ 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix} \ldots$
Coefficient of m^{th} energy eigenstate	$c_m = \langle m \mid \Phi\rangle$	$c_m = \int_0^{2\pi} \sqrt{\frac{1}{2\pi r_0}}e^{-im\phi}\,\Phi(\phi)\,r_0 d\phi$	$\begin{pmatrix} \cdots & 1 & \cdots & 0 & \cdots \end{pmatrix} \begin{pmatrix} \vdots \\ c_m \\ \vdots \\ c_0 \\ \vdots \end{pmatrix}$
Probability of measuring E_m	$P(E_m) = \lvert c_{+m}\rvert^2 + \lvert c_{-m}\rvert^2$ $= \lvert\langle +m \mid \Phi\rangle\rvert^2 + \lvert\langle -m \mid \Phi\rangle\rvert^2$ $m \neq 0$	$P(E_m) = \left\lvert \int_0^{2\pi} \sqrt{\frac{1}{2\pi r_0}}e^{-im\phi}\,\Phi(\phi)\,r_0 d\phi \right\rvert^2$ $+ \left\lvert \int_0^{2\pi} \sqrt{\frac{1}{2\pi r_0}}e^{im\phi}\,\Phi(\phi)\,r_0 d\phi \right\rvert^2$	$P(E_m) = \left\lvert \begin{pmatrix} \cdots & 1 & \cdots & 0 & \cdots \end{pmatrix} \begin{pmatrix} \vdots \\ c_m \\ \vdots \\ c_0 \\ \vdots \end{pmatrix} \right\rvert^2 + \left\lvert \begin{pmatrix} \cdots & 0 & \cdots & 1 & \cdots \end{pmatrix} \begin{pmatrix} \vdots \\ c_0 \\ \vdots \\ c_{-m} \\ \vdots \end{pmatrix} \right\rvert^2$
Expectation value of Hamiltonian	$\langle \Phi \mid H \mid \Phi\rangle = \sum_m \lvert c_m\rvert^2 E_m$	$\langle \Phi \mid H \mid \Phi\rangle = -\int_0^{2\pi} \Phi^*(\phi)\ \frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial\phi^2}\Phi(\phi)\,r_0 d\phi$	$\langle \Phi \mid H \mid \Phi\rangle = \begin{pmatrix} \cdots & c_1^* & c_0^* & c_{-1}^* & \cdots \end{pmatrix} \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & ⋰ \\ \cdots & E_1 & 0 & 0 & \cdots \\ \cdots & 0 & E_0 & 0 & \cdots \\ \cdots & 0 & 0 & E_{-1} & \cdots \\ ⋰ & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ c_1 \\ c_0 \\ c_{-1} \\ \vdots \end{pmatrix}$