Rigid Rotor/Particle on a Sphere

	Ket Representation	Wave Function Representation	Matrix Representation
Hamiltonian	$\hat{H}$	$\hat{H}=\frac{1}{2I}L^2=-\frac{\hbar^2}{2I}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{d^2}{d\phi^2}\right)$	$\begin{pmatrix} E_{0,0} & 0 & 0 & 0 & 0 & \cdots \\ 0 & E_{1,1} & 0 & 0 & 0 & \cdots \\ 0 & 0 & E_{1,0} & 0 & 0 & \cdots \\ 0 & 0 & 0 & E_{1,-1} & 0 & \cdots \\ 0 & 0 & 0 & 0 & E_{2,2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 2\frac{\hbar^2}{2I} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 2\frac{\hbar^2}{2I} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 2\frac{\hbar^2}{2I} & 0 & \cdots \\ 0 & 0 & 0 & 0 & 6\frac{\hbar^2}{2I} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenvalues of Hamiltonian	$E_\ell=\frac{\hbar^2}{2I}\ell(\ell+1)$	$E_\ell=\frac{\hbar^2}{2I}\ell(\ell+1)$	$E_\ell=\frac{\hbar^2}{2I}\ell(\ell+1)$
Normalized Eigenstates of Hamiltonian	$\lvert\ell,m\rangle$	$Y_\ell^m(\theta,\phi)=(-1)^{(m+\lvert m\rvert)/2}\sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-\lvert m\rvert)!}{(\ell+\lvert m\rvert)!}}P_\ell^m(\cos\theta)e^{im\phi}$	$\lvert 0,0\rangle \doteq \begin{pmatrix}1\\0\\0\\0\\\vdots\end{pmatrix}, \lvert 1,1\rangle \doteq \begin{pmatrix}0\\1\\0\\0\\\vdots\end{pmatrix}, \lvert 1,0\rangle \doteq \begin{pmatrix}0\\0\\1\\0\\\vdots\end{pmatrix}, \lvert 1,-1\rangle \doteq \begin{pmatrix}0\\0\\0\\1\\\vdots\end{pmatrix}, \ \ldots$
Coefficient of the energy eigenstate with quantum numbers ℓ,m	$c_{\ell,m}=\langle\ell,m\vert\Psi\rangle$	$c_{\ell,m}=\int_0^{2\pi}\int_0^{\pi}Y_\ell^m(\theta,\phi)^*\Psi(\theta,\phi)\sin\theta d\theta d\phi$	$\begin{pmatrix}\cdots & 1 & \cdots\end{pmatrix}\begin{pmatrix}\vdots\\c_{\ell,m}\\\vdots\end{pmatrix}$
Probability of measuring $E_{\ell,m}$	$\mathcal{P}\left(\frac{\hbar^2}{2I}\ell(\ell+1)\right)=\sum_{m=-\ell}^{\ell}\lvert c_{\ell,m}\rvert^2 = \sum_{m=-\ell}^{\ell}\lvert\langle\ell,m\vert\Psi\rangle\rvert^2$	$\mathcal{P}\left(\frac{\hbar^2}{2I}\ell(\ell+1)\right)=\sum_{m=-\ell}^{\ell}\left\lvert\int_0^{2\pi}\int_0^{\pi}Y_\ell^m(\theta,\phi)^*\Psi(\theta,\phi)\sin\theta d\theta d\phi\right\rvert^2$	$\mathcal{P}(E_\ell)=\sum_{m=-\ell}^{\ell}\left\lvert\begin{pmatrix}\cdots & 1 & \cdots\end{pmatrix}\begin{pmatrix}\vdots\\c_{\ell,m}\\\vdots\end{pmatrix}\right\rvert^2$

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	Ket Representation	Wave Function Representation	Matrix Representation
Operator for square of the angular momentum	L^2	$L^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{d^2}{d\phi^2}\right)$ $= -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\hbar^2\sin^2\theta}L_z{}^2\right)$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 2\hbar^2 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 2\hbar^2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 2\hbar^2 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 6\hbar^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenvalues of L^2	$\hbar^2\ell(\ell+1)$	$\hbar^2\ell(\ell+1)$	$\hbar^2\ell(\ell+1)$
Normalized Eigenstates of L^2	$\|\ell,m\rangle$	$Y_\ell^m(\theta,\phi) = (-1)^{(m+\|m\|)/2}\sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-\|m\|)!}{(\ell+\|m\|)!}}P_\ell^m(\cos\theta)e^{im\phi}$	$\|0,0\rangle \doteq \begin{pmatrix}1\\0\\0\\0\\\vdots\end{pmatrix}, \|1,1\rangle \doteq \begin{pmatrix}0\\1\\0\\0\\\vdots\end{pmatrix}, \|1,0\rangle \doteq \begin{pmatrix}0\\0\\1\\0\\\vdots\end{pmatrix}, \|1,-1\rangle \doteq \begin{pmatrix}0\\0\\0\\1\\\vdots\end{pmatrix}, \quad \ldots$
Coefficient of the eigenstates of L^2 with quantum numbers ℓ, m	$c_{\ell,m} = \langle \ell,m\|\Psi\rangle$	$c_{\ell,m} = \int_0^{2\pi}\int_0^{\pi} Y_\ell^m(\theta,\phi)^*\,\Psi(\theta,\phi)\sin\theta\, d\theta\, d\phi$	$\begin{pmatrix}\cdots & 1 & \cdots\end{pmatrix}\begin{pmatrix}\vdots\\ c_{\ell,m}\\ \vdots\end{pmatrix}$
Probability of measuring $\hbar^2\ell(\ell+1)$ for the square of the angular momentum	$\mathcal{P}\left(\hbar^2\ell(\ell+1)\right) = \sum_{m=-\ell}^{\ell}\left\|c_{\ell,m}\right\|^2$ $= \sum_{m=-\ell}^{\ell}\left\|\langle \ell,m\|\Psi\rangle\right\|^2$	$\mathcal{P}\left(\hbar^2\ell(\ell+1)\right) = \sum_{m=-\ell}^{\ell}\left\|\int_0^{2\pi}\int_0^{\pi} Y_\ell^m(\theta,\phi)^*\,\Psi(\theta,\phi)\sin\theta\, d\theta\, d\phi\right\|^2$	$\mathcal{P}(E_\ell) = \sum_{m=-\ell}^{\ell}\left\|\begin{pmatrix}\cdots & 1 & \cdots\end{pmatrix}\begin{pmatrix}\vdots\\ c_{\ell,m}\\ \vdots\end{pmatrix}\right\|^2$

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	Ket Representation	**Wave Function Representation**	**Matrix Representation**
Operator for z-component of angular momentum	L_z	$-i\hbar\frac{\partial}{\partial\phi}$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \hbar & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & -\hbar & 0 & \cdots \\ 0 & 0 & 0 & 0 & 2\hbar & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenstates of L_z	$m\hbar$	$m\hbar$	$m\hbar$
Normalized Eigenstates of L_z	$\lvert \ell, m\rangle$	$Y_\ell^m(\theta,\phi) = (-1)^{(m+\lvert m\rvert)/2}\sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-\lvert m\rvert)!}{(\ell+\lvert m\rvert)!}}P_\ell^m(\cos\theta)e^{im\phi}$	$\lvert 0,0\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \lvert 1,1\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \lvert 1,0\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}, \lvert 1,-1\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix}, \quad \ldots$
Coefficient of m^{th} eigenstates of L_z	$c_{\ell,m} = \langle \ell, m \lvert \Psi\rangle$	$c_{\ell,m} = \int_0^{2\pi}\int_0^{\pi} Y_\ell^m(\theta,\phi)^* \Psi(\theta,\phi)\sin\theta d\theta d\phi$	$\begin{pmatrix} \cdots & 1 & \cdots \end{pmatrix}\begin{pmatrix} \vdots \\ c_{\ell,m} \\ \vdots \end{pmatrix}$
Probability of measuring $m\hbar$ for z-component of angular momentum	$\mathcal{P}(m\hbar) = \sum_{\ell=\lvert m\rvert}^{\infty} \lvert c_{\ell,m}\rvert^2$ $= \sum_{\ell=\lvert m\rvert}^{\infty} \lvert\langle \ell, m \lvert \Psi\rangle\rvert^2$	$\mathcal{P}(m\hbar) = \sum_{\ell=\lvert m\rvert}^{\infty}\left\lvert \int_0^{2\pi}\int_0^{\pi} Y_\ell^m(\theta,\phi)^* \Psi(\theta,\phi)\sin\theta d\theta d\phi\right\rvert^2$	$\mathcal{P}(m\hbar) = \sum_{\ell=\lvert m\rvert}^{\infty}\left\lvert \begin{pmatrix} \cdots & 1 & \cdots \end{pmatrix}\begin{pmatrix} \vdots \\ c_{\ell,m} \\ \vdots \end{pmatrix}\right\rvert^2$