Math

Total differentials

$$dA = \left(\frac{\partial A}{\partial B}\right)_C dB + \left(\frac{\partial A}{\partial C}\right)_B dC$$

You can:

1. Do algebra
2. Interpret coefficients as partial derivatives
3. Integrate

Mixed partial derivatives

$$\left(\frac{\partial \left(\frac{\partial A}{\partial B}\right)_C}{\partial C}\right)_B = \left(\frac{\partial \left(\frac{\partial A}{\partial C}\right)_B}{\partial B}\right)_C$$

Chain rules

$$\left(\frac{\partial A}{\partial B}\right)_C = \frac{1}{\left(\frac{\partial B}{\partial A}\right)_C}$$

$$\left(\frac{\partial A}{\partial B}\right)_D = \left(\frac{\partial A}{\partial C}\right)_D \left(\frac{\partial C}{\partial B}\right)_D$$

$$\left(\frac{\partial A}{\partial B}\right)_C = -\frac{\left(\frac{\partial A}{\partial C}\right)_B}{\left(\frac{\partial B}{\partial C}\right)_A}$$

Thermodynamics

Entropy

$$\Delta S = \int đ\frac{Q_{\text{quasistatic}}}{T}$$

$$đQ = TdS$$

$$C_\alpha = T\left(\frac{\partial S}{\partial T}\right)_\alpha$$

First Law

$$\Delta U = Q + W$$

$$dU = đQ + đW$$

$$dU = TdS - pdV$$

Second Law

$$\Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \geq 0$$

Legendre transforms

You can add or subtract from U products of conjugate variables to find new thermodynamic potentials that are convenient when T or p are held fixed or controlled.

Maxwell relations

From any thermodynamic potential you can use the equality of mixed partial derivatives to create a relationship between two different partial derivatives.

Statistical mechanics

$$P_i = \frac{e^{-\beta E_i}}{Z}$$

$$Z = \sum_i^{\text{all states}} e^{-\beta E_i}$$

$$\beta = \frac{1}{k_B T}$$

$$F = -k_B T \ln Z$$

$$U = \sum_i^{\text{all states}} P_i E_i$$

$$S = -k_B \sum_i^{\text{all states}} P_i \ln P_i$$