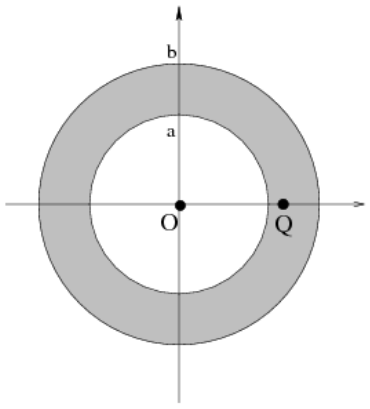
1 Gravitational Field and Mass

The gravitational field due to a spherical shell of matter (or equivalently, the electric field due to a spherical shell of charge) is given by:

$$\vec{g} = \begin{cases} 0 & \text{for } r < a \\ -G\,\frac{M}{b^3 - a^3}\left(r - \frac{a^3}{r^2}\right)\hat{r} & \text{for } a < r < b \\ -G\,\frac{M}{r^2}\,\hat{r} & \text{for } r > b \end{cases} \tag{1}$$



This problem explores the consequences of the divergence theorem for this shell.

(a) Using the given description of the gravitational field, find the divergence of the gravitational field everywhere in space. You will need to divide this question up into three parts: $r < a$, $a < r < b$, and $r > b$.

(b) Briefly discuss the physical meaning of the divergence in this particular example.

(c) For this gravitational field, verify the divergence theorem on a sphere, concentric with the shell, with radius Q, where $a < Q < b$. ("Verify" the divergence theorem means calculate the integrals from both sides of the divergence theorem and show that they give the same answer.)

(d) Briefly discuss how this example would change if you were discussing the electric field of a uniformly charged spherical shell.