1 Quantum Particle in a 2-D Box

You know that the normalized spatial eigenfunctions for a particle in a 1-D box of length L are $\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$. If you want the eigenfunctions for a particle in a 2-D box, then you just *multiply* together the eigenfunctions for a 1-D box in each direction. (This is what the separation of variables procedure tells you to do.)

(a) Find the normalized eigenfunctions for a particle in a 2-D box with sides of length L_x in the x-direction and length L_y in the y-direction.

(b) Find the Hamiltonian for a 2-D box and show that your eigenstates are indeed eigenstates and find a formula for the possible energies

(c) Any sufficiently smooth spatial wave function inside a 2-D box can be expanded in a **double** sum of the product wave functions, i.e.

$$\psi(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{nm} \text{ eigenfunction}_n(x) \text{ eigenfunction}_m(y) \tag{1}$$

Using your expressions from part (a) above, write out all the terms in this sum out to $n = 3$, $m = 3$. Arrange the terms, conventionally, in terms of increasing energy.

You may find it easier to work in bra/ket notation:

$$\begin{aligned} |\psi\rangle &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{nm} |n\rangle |m\rangle \\ &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{nm} |nm\rangle \end{aligned}$$

(d) Find a formula for the c_{nm}s in part (b). Find the formula first in bra ket notation and then rewrite it in wave function notation.