1 Completeness Relation Change of Basis

(a) Given the polar basis kets written as a superposition of Cartesian kets

$$\begin{aligned} |\hat{s}\rangle &= \cos\phi\,|\hat{x}\rangle + \sin\phi\,|\hat{y}\rangle \\ \left|\hat{\phi}\right\rangle &= -\sin\phi\,|\hat{x}\rangle + \cos\phi\,|\hat{y}\rangle \end{aligned}$$

Find the following quantities:

$$\langle\hat{x}|\hat{s}\rangle\,, \quad \langle\hat{y}|\hat{s}\rangle\,, \quad \left\langle\hat{x}\middle|\hat{\phi}\right\rangle, \quad \left\langle\hat{y}\middle|\hat{\phi}\right\rangle$$

(b) Given a vector written in the polar basis

$$|\vec{v}\rangle = a\,|\hat{s}\rangle + b\left|\hat{\phi}\right\rangle$$

where a and b are known. Find coefficients c and d such that

$$|\vec{v}\rangle = c\,|\hat{x}\rangle + d\,|\hat{y}\rangle$$

Do this by using the completeness relation:

$$|\hat{x}\rangle\langle\hat{x}| + |\hat{y}\rangle\langle\hat{y}| = 1$$

(c) Using a completeness relation, change the basis of the spin-1/2 state

$$|\Psi\rangle = g\,|+\rangle + h\,|-\rangle$$

into the S_y basis. In otherwords, find j and k such that

$$|\Psi\rangle = j\,|+\rangle_y + k\,|-\rangle_y$$