

1 Paramagnet (multiple solutions)

We have the following equations of state for the *total magnetization* M , and the entropy S of a paramagnetic system:

$$M = N\mu \frac{e^{\frac{\mu B}{k_B T}} - e^{-\frac{\mu B}{k_B T}}}{e^{\frac{\mu B}{k_B T}} + e^{-\frac{\mu B}{k_B T}}} \tag{1}$$

$$S = Nk_B \left\{ \ln 2 + \ln \left(e^{\frac{\mu B}{k_B T}} + e^{-\frac{\mu B}{k_B T}} \right) + \frac{\mu B}{k_B T} \frac{e^{\frac{\mu B}{k_B T}} - e^{-\frac{\mu B}{k_B T}}}{e^{\frac{\mu B}{k_B T}} + e^{-\frac{\mu B}{k_B T}}} \right\} \tag{2}$$

(a) List variables in their proper positions in the middle columns of the charts below.

(b) Solve for the *magnetic susceptibility*, which is defined as:

$$\chi_B = \left(\frac{\partial M}{\partial B} \right)_T$$

(c) Using **both** the differentials (zapping with d) and chain rule diagram methods, find a chain rule for:

$$\left(\frac{\partial M}{\partial B} \right)_S$$

(d) Evaluate your chain rule. Sense-making: Why does this come out to zero?

Prompt	Variable Positions in Notation	Variables	Variable Categories
Partial derivative in Leibniz notation	Numerator Variable $\left(\frac{\partial \blacksquare}{\partial \blacksquare} \right)_{\blacksquare}$		Dependent Variable (function to be differentiated)
$\left(\frac{\partial M}{\partial B} \right)_T$	Denominator Variable $\left(\frac{\partial \blacksquare}{\partial \blacksquare} \right)_{\blacksquare}$		Independent Variable (differentiated with respect to)
	Subscript Variable $\left(\frac{\partial \blacksquare}{\partial \blacksquare} \right)_{\blacksquare}$		Extra Independent Variable (held constant)
Given system of equations	Absent Variable $\left(\frac{\partial \blacksquare}{\partial \blacksquare} \right)_{\blacksquare}$		Extra Dependent Variable (reexpressed in terms of other variables or differentials)
$M(B, T)$ $S(B, T)$			

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Partial derivative in Leibniz notation $\left(\frac{\partial M}{\partial B}\right)_S$	Numerator Variable $\left(\frac{\partial \blacksquare}{\partial \blacksquare}\right)_{\blacksquare}$		Dependent Variable (function to be differentiated)
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Given system of equations $M(B, T)$ $S(B, T)$	Subscript Variable $\left(\frac{\partial \blacksquare}{\partial \blacksquare}\right)_{\blacksquare}$		Extra Independent Variable (held constant)
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