

1 Energy, Entropy, and Probabilities

The goal of this problem is to show that once we have maximized the entropy and found the microstate probabilities in terms of a Lagrange multiplier β , we can prove that $\beta = \frac{1}{kT}$ based on the statistical definitions of energy and entropy and the thermodynamic definition of temperature embodied in the thermodynamic identity.

The internal energy and entropy are each defined as a weighted average over microstates:

$$U = \sum_i E_i P_i \qquad S = -k_B \sum_i P_i \ln P_i \qquad (1)$$

We saw in class that the probability of each microstate can be given in terms of a Lagrange multiplier β as

$$P_i = \frac{e^{-\beta E_i}}{Z} \qquad Z = \sum_i e^{-\beta E_i} \qquad (2)$$

Put these probabilities into the above weighted averages in order to relate U and S to β . Then make use of the thermodynamic identity

$$dU = TdS - pdV \qquad (3)$$

to show that $\beta = \frac{1}{kT}$.