1 Magnetic susceptibility

Consider a paramagnet, which is a material with n spins per unit volume each of which may each be either "up" or "down". The spins have energy $\pm mB$ where m is the magnetic dipole moment of a single spin, and there is no interaction between spins. The magnetization M is defined as the total magnetic moment divided by the total volume. *Hint:* each individual spin may be treated as a two-state system, which you have already worked with above.

(a) Find the Helmholtz free energy of a paramagnetic system (assume N total spins) and show that $\frac{F}{NkT}$ is a function of only the ratio $x \equiv \frac{mB}{kT}$.

(b) Use the canonical ensemble (i.e. partition function and probabilities) to find an exact expression for the total magentization M (which is the total dipole moment per unit volume) and the susceptibility

$$\chi \equiv \left(\frac{\partial M}{\partial B}\right)_T \tag{1}$$

as a function of temperature and magnetic field for the model system of magnetic moments in a magnetic field. The result for the magnetization is

$$M = nm \tanh\left(\frac{mB}{kT}\right) \tag{2}$$

where n is the number of spins per unit volume. The figure shows what this magnetization looks like.

(c) Show that the susceptibility is $\chi = \frac{nm^2}{kT}$ in the limit $mB \ll kT$.

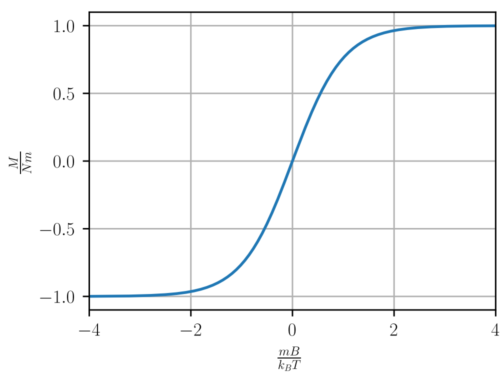

Figure 1: Plot of magnetization vs. B field