

# 1 Paramagnetism

Find the equilibrium value at temperature  $T$  of the fractional magnetization

$$\frac{\mu_{tot}}{Nm} \equiv \frac{2\langle s \rangle}{N} \quad (1)$$

of a system of  $N$  spins each of magnetic moment  $m$  in a magnetic field  $B$ . The spin excess is  $2s$ . The energy of this system is given by

$$U = -\mu_{tot}B \quad (2)$$

where  $\mu_{tot}$  is the total magnetization. Take the entropy as the logarithm of the multiplicity  $g(N, s)$  as given in (1.35 in the text):

$$S(s) \approx k_B \log g(N, 0) - k_B \frac{2s^2}{N} \quad (3)$$

for  $|s| \ll N$ , where  $s$  is the spin excess, which is related to the magnetization by  $\mu_{tot} = 2sm$ . *Hint:* Show that in this approximation

$$S(U) = S_0 - k_B \frac{U^2}{2m^2 B^2 N}, \quad (4)$$

with  $S_0 = k_B \log g(N, 0)$ . Further, show that  $\frac{1}{kT} = -\frac{U}{m^2 B^2 N}$ , where  $U$  denotes  $\langle U \rangle$ , the thermal average energy.