1 Gibbs sum for a two level system

(a) Consider a system that may be unoccupied with energy zero, or occupied by one particle in either of two states, one of energy zero and one of energy ε. Find the Gibbs sum for this system is in terms of the activity $\lambda \equiv e^{\beta\mu}$. Note that the system can hold a maximum of one particle.

(b) Solve for the thermal average occupancy of the system in terms of λ.

(c) Show that the thermal average occupancy of the state at energy ε is

$$\langle N(\varepsilon)\rangle = \frac{\lambda e^{-\frac{\varepsilon}{kT}}}{\mathcal{Z}} \tag{1}$$

(d) Find an expression for the thermal average energy of the system.

(e) Allow the possibility that the orbitals at 0 and at ε may each be occupied each by one particle at the same time; Show that

$$\mathcal{Z} = 1 + \lambda + \lambda e^{-\frac{\varepsilon}{kT}} + \lambda^2 e^{-\frac{\varepsilon}{kT}} \tag{2}$$

$$= (1+\lambda)\left(1 + e^{-\frac{\varepsilon}{kT}}\right) \tag{3}$$

Because $\mathcal{Z}$ can be factored as shown, we have in effect two independent systems.