1 Photon carnot engine

In our week on radiation, we saw that the Helmholtz free energy of a box of radiation at temperature T is

$$F = -8\pi \frac{V(kT)^4}{h^3c^3}\frac{\pi^4}{45} \tag{1}$$

From this we also found the internal energy and entropy

$$U = 24\pi \frac{(kT)^4}{h^3c^3}\frac{\pi^4}{45}V \tag{2}$$

$$S = 32\pi kV\left(\frac{kT}{hc}\right)^3\frac{\pi^4}{45} \tag{3}$$

Given these results, let us consider a Carnot engine that uses an empty metalic piston (i.e. a photon gas).

(a) Given T_H and T_C, as well as V_1 and V_2 (the two volumes at T_H), determine V_3 and V_4 (the two volumes at T_C).

(b) What is the heat Q_H taken up and the work done by the gas during the first isothermal expansion? Are they equal to each other, as for the ideal gas?

(c) Does the work done on the two isentropic stages cancel each other, as for the ideal gas?

(d) Calculate the total work done by the gas during one cycle. Compare it with the heat taken up at T_H and show that the energy conversion efficiency is the Carnot efficiency.