

1 Euler's Formula II

- (a) Look up or derive the Taylor series expansions for $\cos \theta$, $\sin \theta$, and e^x .
- (b) Show that $e^{i\theta} = \cos \theta + i \sin \theta$.
- (c) Verify that $e^{2i\theta} = (e^{i\theta})^2$ using trigonometric identities, that is, show that

$$\cos 2\theta + i \sin 2\theta = (\cos \theta + i \sin \theta)^2$$

2 The Rotation Group $SO(2)$

Consider rotation matrices of the form

$$M(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

- (a) Show that
 - (a) $M(0) = 1$
 - (b) $M(-\theta) = M(\theta)^{-1}$
 - (c) $M(\alpha + \beta) = M(\alpha)M(\beta)$
- (b) Compute $A = \dot{M} = \frac{dM}{d\theta} \Big|_{\theta=0}$.
- (c) Show that $M(\theta) = \exp(A\theta)$.