1 Matrix Differential Equation I

Let $P = \begin{pmatrix} i\alpha & 0 \\ 0 & 0 \end{pmatrix}$, $Q = \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix}$, $v(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$, and $c = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$.

(a) Solve the differential equation $\frac{dv}{dt} = Pv$ with initial conditions $v(0) = c$.

(b) Solve the differential equation $\frac{dv}{dt} = Qv$ with initial conditions $w(0) = c$.

For full credit, your solutions should involve matrix exponentiation.

2 Matrix Differential Equation II

Let $A = \begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix}$, $v(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$, and $v_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$. Solve the differential equation $\frac{dv}{dt} = Av$ with initial conditions $v(0) = v_0$.

HINT: $e^{At} = \begin{pmatrix} \cosh(\alpha t) & \sinh(\alpha t) \\ \sinh(\alpha t) & \cosh(\alpha t) \end{pmatrix}$.