# 1 Dynkin diagrams

The goal of this problem is to construct all possible Dynkin diagrams for simple Lie algebras. For the purposes of this problem, a Dynkin diagram has the following properties:

- There are $n$ points, representing simple roots.
- Two points may be connected by 0, 1, 2, or 3 lines.
- The diagram is connected, that is, there is a path from every point to every other point.

Furthermore, Dynkin diagrams obey the following rules:

- There exist at most $k-1$ connections (each consisting of 1, 2, or 3 lines) among $k$ points.
- There exist at most 3 lines at any point.
- If a simple chain ($k$ points connected by single lines) in a valid diagram is replaced by a single point, the resulting diagram is also valid.
- If a simple chain of length $p$ is connected by a double line to a simple chain of length $q$, then $(p-1)(q-1) < 2$.
- If three simple chains of lengths $p$, $q$, and $r$ meet at a single point (which counts as belonging to all three chains), then $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} > 1$.

Find all Dynkin diagrams that satisfy these constraints.