1 Human Vision

(Q4M.5 from textbook) Suppose you are standing in the dark and facing a 20 W LED bulb 100 m away. If the diameter of your pupils is about 8 mm under these conditions, about how many photons of *visible* light enter your eye every second?

2 Hot hydrogen atom

Find the wavelength of the photon emitted during a $n = 5 \rightarrow 4$ transition in a hydrogen atom.

Note: The energy levels in a hydrogen atom are

$$E_n = \frac{-13.6 \text{ eV}}{n^2} \tag{1}$$

where $n = 1,\ 2,\ 3,\ ...$

3 Wavelength from a charge on a spring

Suppose a charged particle is held in position by an electrostatic spring (i.e. the restoring force on the charge follows Hooke's law, $F = -kx$). The mass of the charge, and the spring constant, are such that the system has a natural frequency $\omega = 10^{16}$ rad/s (ω is a fixed parameter in this question, not a variable). Find the wavelength of the photon emitted during a $n = 1 \rightarrow 0$ transition.

By solving the Schrodinger equation for this situation, we know that the energy of the charged particle (i.e. the sum of the particle's kinetic energy, plus any potential energy stored in the spring) is given by

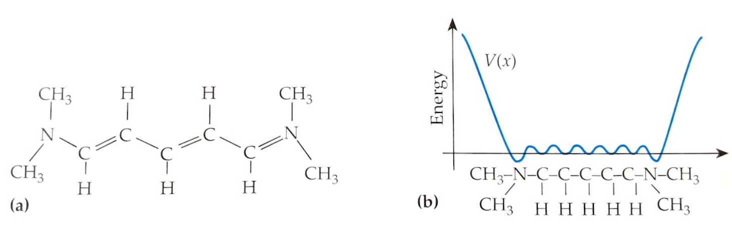
$$E_n = \hbar\omega(n + \frac{1}{2}) \tag{2}$$

where $n = 0,\ 1,\ 2,\ ...$

Note: Due to the shape/symmetry of the wavefunctions for particles trapped by a springlike force, optical transitions only occur when $\Delta n = \pm 1$.

Sense making: Try approaching this question from a classical physics perspective. What wavelength of light would we expect from a charge that oscillates at $\omega = 10^{16}$ rad/s?

4 Light from an electron in a box



Suppose an electron is trapped in a box whose length is $L = 1.2$ nm. This is a coarse-grained model for an electron in a small molecule like cyanine (see Example Q11.1 in the textbook, and the figure above). If we solve the Schrodinger equation for this coarse-grained model, the possible energy levels for this electron are

$$E = \frac{h^2 n^2}{8mL^2} \tag{3}$$

where m is the mass of the electron and $n = 1,\ 2,\ 3,\ \ldots$

Draw a spectrum chart (like the righthand side of Figure Q11.2) to show what you would see if a number of identical excited systems of this type emitted light that was dispersed by a diffraction grating.

Note: Due to the shape/symmetries of electron wavefunctions in a box, optical transitions between energy levels only happen when $\Delta n = n_{\text{initial}} - n_{\text{final}}$, is an odd integer.

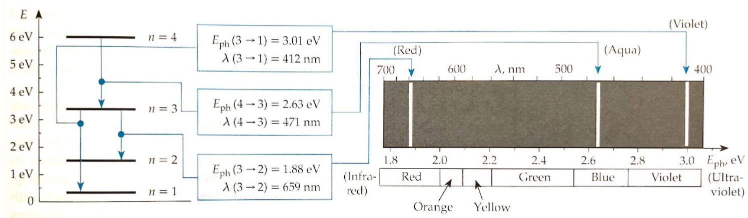
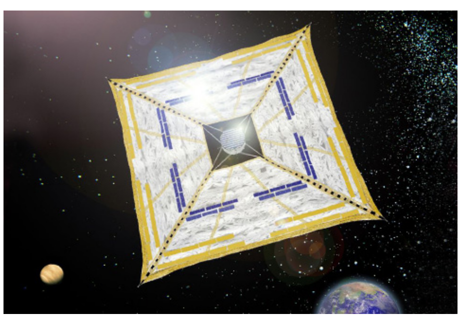


Figure Q11.2
The transitions of an electron in a 1.0-nm box that give rise to visible photons. The spectrum shown on the right is what you would see if a number of identical excited systems of this type emitted light that was dispersed by a prism.

5 Solar Sail



The first spacecraft using a solar sail for propulsion was launched in 2010. Its name is IKAROS. It has a square sail with dimensions 14 m x 14 m. Assume that the sail's mass is 2 kg and it reflects 100% of incident photons. When IKAROS is loaded with other equipment, the total mass of the vehicle is 10 kg. The sail is orientated to receive maximum light from the sun.

(a) Calculate the momentum of the photons that come from the sun and hit the solar sail in 1 second. Assume a solar intensity of 1300 J/(s.m^2).

(b) How much momentum will be transferred from solar photons to IKAROS in one day? Give a numerical answer in units of kg.m/s (assume a constant solar intensity).

(c) What is the change in the solar sail's velocity in one day? (assume that acceleration is only caused by sunlight).