1 Cosmic Background Radiation

The universe is filled with thermal radiation that has a blackbody spectrum at an effective temperature of 2.7 K. What is the wavelength of light that corresponds to the peak in S_λ (S_λ is the spectral distribution with respect to wavelength)? In what region of the electromagnetic spectrum is this peak wavelength?

2 Efficiency of a solar cell

Pick between: the solar cell efficiency question, or the space shuttle landing question.

	A	B
1	280	0.145962
2	285	0.276177
3	290	0.477193
4	295	0.5355
5	300	0.496948
6	305	0.592171
7	310	0.640309
8	315	0.690968
9	320	0.731223
10	325	0.849888
11	330	1.00566
12	335	0.941141
13	340	0.967548

Download the file extraterr_solar.csv, which is in comma-separated-variable (csv) format. Open the csv file in a spreadsheet program such as Excel. The data is the spectral intensity with respect to wavelength, S_λ, for the sunlight that is hitting a satellite above the earth. The first column is wavelength in units of nanometers. The second column is spectral intensity in units of W/(m^2·nm).

(a) Use a spreadsheet to perform a simple numerical integration (Riemann sum) to find the total energy flux hitting the satellite. Explain your method using summation notation. Additionally, write down the formula you enter in the spreadsheet (e.g. =SUM(B1:B745)). Give your final answer in units of W/m^2 and check that it is reasonable.

(b) Consider a narrow band of wavelengths, from 552.5 nm to 557.5 nm. (The bandwidth is 5 nm and the central wavelength is 555 nm). All the photons in this bandwidth have very similar energy, $E_{\text{photon}} \approx$ (1240 nm·eV)/(555 nm). How many photons per second per m^2 are in this spectral band of sunlight? Explain your method using standard mathmeatical notation. Additionally, write down the formula that you entered into the spreadsheet.

The calculation that you did for part b can now be applied to every row in your spreadsheet. You will need these numbers for part c.

(c) Silicon solar cells absorb photons if $E_{\text{photon}} > 1.1$ eV. That is to say, E_{photon} must be greater than gap between occupied and unoccupied quantum energy levels in silicon. Use your spreadsheet to calculate how many photons per second per m^2 have sufficient energy to be absorbed by a solar cell. Write down the formula that you entered into the spreadsheet.

(d) The electrical energy produced by a silicon solar cell cannot exceed (1.1 eV) $\times$ (number of absorbed photons Calculate the maximum possible rate that electrical energy could be produced by a solar cell attached to this satellite per unit area. Give your answer in units of W/m^2.

(e) Compare your answers to part a and part d. What is the maximum possible efficiency of the solar cell (i.e. the ratio of the electrical energy output to the total energy input)?

3 Two-layer model for estimating the Earth's temperature

Two layers of plexiglass are surrounding the Earth. One layer is 5 km above sea level, the other layer is 10 km above sea level. These plexiglass layers have replaced the gaseous atmosphere. Both layers of plexiglass are transparent to the solar spectrum (wavelengths centered around 500 nm), but fully absorb the thermal radiation emitted from the surface of the Earth. The surface of the Earth absorbs 70% of the incident sunlight and reflects the rest. Assume that the Earth distributes the absorbed solar energy uniformly across its spherical surface, therefore, it has a uniform temperature, T_{surf}. Every part of this system is in steady state, meaning, all temperatures are stable.

(a) Draw an energy flow diagram for this system with three **boxes** representing (i) the surface of the Earth, (ii) the first plexiglass layer, (iii) the second plexiglass layer. Draw **arrows** to represent energy transported by short-wavelength light (solar radiation centered around 500 nm) and long-wavelength light (earth glow centered around 10,000 nm). If energy is being exchanged in two directions, show this with two separate arrows.

(b) The surface of the earth has temperature T_{surf}, the first plexiglass layer has temperature T_1, and the second plexiglass layer has temperature T_2. Treat these as unknowns (we will determine them in part c). Use the idea of balanced energy rates to write down a set of mathematical relationships relating T_{surf}, T_1 and T_2. Other parameters that may appear in your expressions include:
I_{sun}, intensity of sunlight
R_{earth}, radius of Earth
σ, the Stefan-Boltzmann constant

Note: Remember that the absorption of Sunlight depends on the size of the earth's shadow, not on the surface area of the earth.
Note: The surface area of the plexiglass layers are almost equal to the surface area of the Earth (the difference is negligible).

(c) Let $I_{\text{sun}} = 1360$ J/(s m^2) and solve for T_2, T_1 and T_{surf}. Give your final answer in both kelvin and your preferred unit for describing air temperature.