1 Atmospheric Muons

(Taylor 15.7)

The muons created by cosmic rays in the upper atmosphere rain down more-or-less uniformly on the earth's surface, although some of them decay on the way down, with a half-life of about 1.5 μs (measured in their rest frame). A muon detector is carried in a balloon to an altitude of 2000m and in the course of an hour detects 650 muons traveling at .99c toward the earth. In an identical detector that remains at sea level, how many muons should I register in one hour? Calculate your answer two ways:

(a) Classically (no time dilation)

(b) Relativitically (time dilation)

(Remember that after n half-lives 2^{-n} of the original particles survive.) Needless to say, the relativistic answer agrees with experiment.

Sensemaking: Check that in both the classical and relativistic cases, fewer muons are detected on the ground than in the balloon.

Sensemaking: Does the difference between the classical and relativistic answer make sense? Explain.

2 The Twin Paradox

None

(modified from Griffiths 12.16)

On their 21st birthday, one twin, Alex, gets on a moving sidewalk, which carries her out to star X at speed $4/5c$. Her twin brother, Blake, stays home. When Alex gets to star X, she immediately jumps onto the returning (inbound) moving sidewalk and comes back to earth, again at speed $4/5c$. She arrives on her 39th birthday (as determined by *her* watch).

(a) What is potentially paradoxical about this situation?

(b) You're going to resolve this paradox using a Spacetime diagram and Lorentz transformations. Draw a single spacetime diagram showing the entire trip in the reference frame of Blake. Your diagram should show the world lines of both twins. Label all events and show the space and time axes of the different frames. Update your spacetime diagram as you answer the following questions.

(c) How old is Blake (who stayed at home)?

(d) From Blake's perspective, how far away is star X? (Give your answer in light years.)

Now, we're going to use a trick: we're going to recognize that Alex actually switches reference frames at Star X, and we're going to calibrate this "inbound" reference frame the same as the other two frames. We'll call the outbound sidewalk system S' and the inbound one S'' (the earth system is S). All three frames synchronize their their master clocks and choose their origins at the location of Earth so that all the origins are colocated at $t = t' = t'' = 0$

(e) What are the coordinates (x, ct) of the jump (from the outbound to inbound sidewalk) in S (Blake's frame)?

(f) What are the coordinates (x', ct') of Alex's jump in S' (Alex's outbound frame)?

(g) What are the coordinates (x'', ct'') of the jump in S'' (Alex's inbound frame)?

(h) If Alex wanted her watch to agree with the clock in S'', how would she have to reset it immediately after the jump? (In other words, what is the time of the jump in the S'' frame.) If she *did* this, what would her watch read when she got home? (This wouldn't change her *age*, of course—she's still 39—it would just make her watch agree with the standard synchronization in S''.

(i) If Alex is asked the question, "What is Blake's age on Earth right now?", what is the correct reply:

  (a) just before she makes the jump?

  (b) just after she makes the jump?

  (Hint: You need to consider an event at Blake's location that is simultaneous (from Alex's perspective) with the jump.)

(j) How many earth years does the return trip take? Add this to (ii) from (I) to determine how old *Alex* expects Blake to be at their reunion.

(k) How is the paradox resolved?

3 Angles Aren't Invariant

(*Taylor 15.13*)

A meter stick is at rest in frame S_0, which is traveling with speed $v = 0.8c$ along the x-axis of the frame S.

(a) (a) The stick lies in the x_0, y_0 plane and makes an angle $\theta_0 = 60°$ with the x_0 axis (as measured in S_0. What is the length l as measured in S, and what is its angle θ with the x axis?

  (b) *Sensemaking: Compare Quantities* Compare the length and angle of the stick as observed by S to the length and angle observed by S_0.

(b) (a) For the same relative speed between frames, what is θ_0 when $\theta = 60°$? What is l in this case?

  (b) *Sensemaking: Compare Quantities* How does your answer compare to the situation in part (a)?

4 Time Dilation From Lorentz

(based on Taylor 15.18)

Use the Lorentz transformation to rederive the time-dilation formula. *Sensemaking: Visualize the Situation with a Graph* As part of your solution, draw and label a spacetime diagram of the situation of your calculation.

5 Interval is Frame Invariant

(a) Show that the square interval $s^2 = x^2 - (ct)^2$ is frame invariant. (Hint: do a Lorentz transformation.)

(b) *Sensemaking: Put in Context* Explain the significance of this invariance - what do you think this mean and why do you think it is important?