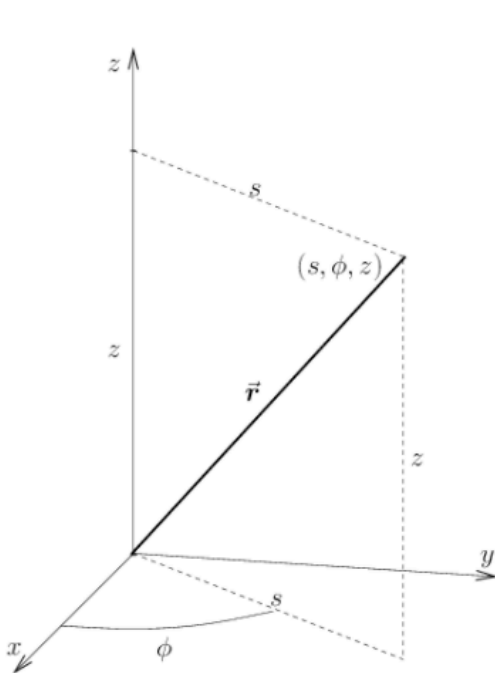


1 Speed Squared in Curvilinear Coordinates

Show that the speed squared written in different standard coordinates systems is:

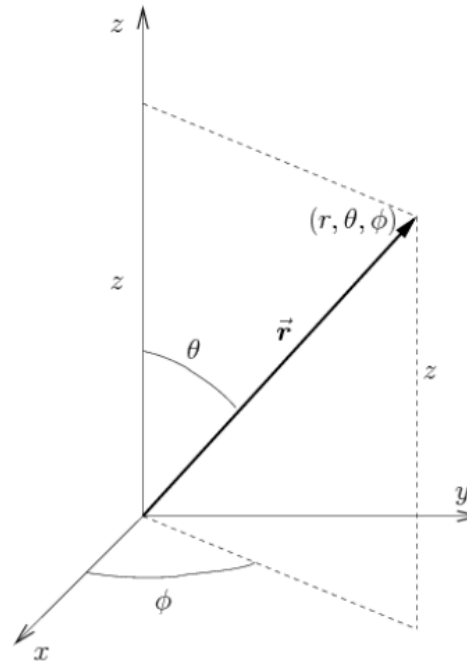
- (a) in rectangular coordinates: $v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$
 (b) in cylindrical coordinates: $v^2 = \dot{s}^2 + s^2\dot{\phi}^2 + \dot{z}^2$
 (c) in spherical coordinates: $v^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2$

where:



Cylindrical Coordinates

$$\begin{aligned}x &= s \cos \phi \\y &= s \sin \phi \\z &= z\end{aligned}$$



Spherical Coordinates

$$\begin{aligned}x &= r \sin \theta \cos \phi \\y &= r \sin \theta \sin \phi \\z &= r \cos \theta\end{aligned}$$

The algebra for these calculations is a little tedious, but it's good to do them once in your career to convince yourself that these expressions are true!

See: GMM: Curvilinear Coordinates for more discussion of curvilinear coordinates. Note that the labels for the azimuthal and polar coordinates in different for math and physics (ϕ and θ are swapped.)