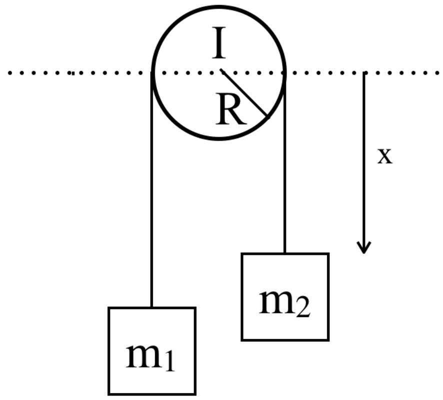
1 Atwood Machine

Consider an Atwood machine, in which two blocks m_1 and m_2 are suspended by an inextensible string (length l) which passes over a pulley with moment of inertia I and radius R. The pulley has frictionless bearings.



(a) Starting from Newton's 2nd Law, find the acceleration of the system.

(b) Write down the Lagrangian of the system and use it to find the acceleration of the system. (Do you get the same answer?)

(c) Use 3 sensemaking strategies to evaluate your answer.

2 Bead on a Rotating Rod

(*modified from Taylor 7.21*)

The center of a long frictionless rod is pivoted at the origin, and the rod is forced to rotate in a horizontal plane with constant angular velocity ω. Consider a bead with mass m that is free to move frictionlessly along the rod. Find the position of the bead as a function of time using the polar coordinate s as your generalized coordinate.

(a) Using a Lagrangian approach, find the position of the bead as a function of time using the polar coordinate s as your generalized coordinate.

(b) Use 3 sensemaking strategies to evaluate your answer. (Be sure to articulate your expected results during each strategy.)

3 Bead on a Helix 1

(adapted from Taylor 7.20)

A smooth wire is bent into the shape of a helix, with cylindrical polar coordinates $s = R$ and $z = \lambda\phi$, where R and λ are constants and z is vertically up (and gravity vertically down). Using z as your generalized coordinate, write down the Lagrangian for a bead of mass m threaded on the wire.

(a) Find the bead's vertical acceleration $\ddot{z}$.

(b) Use three sensemaking strategies to evaluate your answer to part (a). (Don't forget to articulate your expected result for each strategy.)

(c) In the limit that $\lambda \to 0$, what is $\ddot{z}$? Explain conceptually how this makes sense.

(d) How does the acceleration depends on the parameters of the problem (R and λ)? Plot the $\ddot{z}$ vs. R and $\ddot{z}$ vs. λ. Explain what these plots tell you about how the geometry of the helix affects the acceleration.

4 Generalized Force and Generalized Momentum

Demonstrate that when your generalized coordinate is an angle, the generalized force associated with that coordinate is a torque and that the generalized momentum associated with that coordinate is an angular momentum.