# 1 Vector Sketch (Rectangular Coordinates)

*More practice for seeing the shape of vector fields, based on their algebraic form. These examples are in rectangular coordinates. You have some examples in cylindrical and spherical coordinates to turn in.* Sketch each of the vector fields below.

(a) $\vec{\boldsymbol{F}} = -y\,\hat{\boldsymbol{x}} + x\,\hat{\boldsymbol{y}}$

(b) $\vec{\boldsymbol{G}} = x\,\hat{\boldsymbol{x}} + y\,\hat{\boldsymbol{y}}$

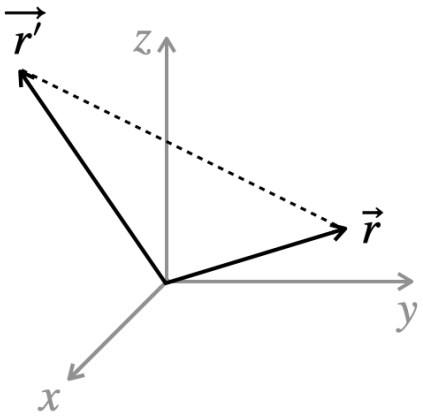
(c) $\vec{\boldsymbol{H}} = y\,\hat{\boldsymbol{x}} + x\,\hat{\boldsymbol{y}}$

# 2 Distance Formula in Curvilinear Coordinates

*This problem was assigned in Theoretical Mechanics. We will use the result in Static Fields. It's here for you to review, especially if you were not in Theoretical Mechanics.*

The distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

(a) Find the distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ in rectangular coordinates.




(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{s^2 + s\,'^2 - 2ss\,'\cos(\phi - \phi\,') + (z - z\,')^2}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{r'^2 + r\,^2 - 2rr\,'\left[\sin\theta\sin\theta\,'\cos(\phi - \phi\,') + \cos\theta\cos\theta\,'\right]}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(d) Now assume that $\vec{r}\,'$ and $\vec{r}$ are in the $x$-$y$ plane. Simplify the previous two formulas.

# 3 Power Series Practice

*Problems like this were covered in Theoretical Mechanics. We will use these techniques in Static Fields. It's here for you to review, especially if you were not in Theoretical Mechanics.*

(a) Calculate the $n = 0, 1, 2, 3, 4$ coefficients of the power series for $\cos z$ expanded around $z = \pi$. Using these coefficients, find a power series approximation for this function.

(b) Plot both the original function and your approximation.

(c) For what values of $z$ is your approximation "good"?

# 4 Series Notation 1

*Try this problem if you need some practice with summation notation for series.*

Write out the first four nonzero terms in the series:

(a)

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

(c)

$$\sum_{n=0}^{\infty} (-2)^n \, \theta^{2n} \tag{1}$$

# 5 Series Notation 2

*Try this problem if you need some practice with summation notation for series.*

Write (a good guess for) the following series using sigma ($\sum$) notation. (If you only know a few terms of a series, you don't know for sure how the series continues.)

(a)

$$1 - 2\,\theta^2 + 4\,\theta^4 - 8\,\theta^6 + \dots$$

(b)

$$\frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \dots$$