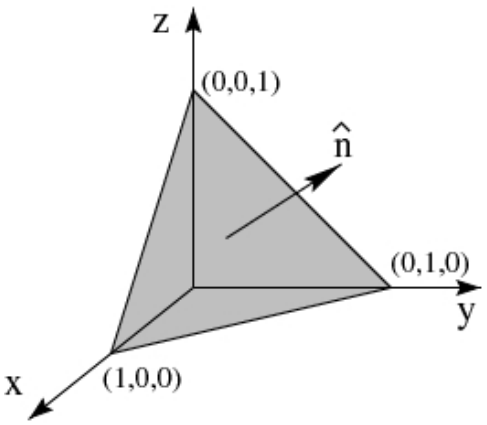

1 Electric Field from a Rod

This is a typical quiz/exam problem. Consider a thin charged rod of length L standing along the z-axis with the bottom end on the xy-plane. The charge density λ is constant. Find the electric field at the point $(0, 0, 2L)$.

2 Electric Field Due to a Ring - Limiting Cases

If you need more practice with series problems. In class, we considered the electric field due to a charged ring with total charge Q and radius R.

Find the first two non-zero terms of a series expansion for the electric field at each of these special locations:

(a)

$$\vec{E}(\vec{r}) = \frac{kQ}{2\pi} \int \frac{\left((s\cos\phi - R\cos\phi')\,\hat{x} + (s\sin\phi - R\sin\phi')\,\hat{y} + z\hat{z}\right)}{\left(s^2 + R^2 - 2sR\cos(\phi - \phi') + z^2\right)^{\frac{3}{2}}} d\phi' \tag{1}$$

Near the center of the ring, on the axis perpendicular to the plane of the ring;

(b) Far from the ring, in the plane of the ring;

3 Cross Triangle

Use the cross product to find the components of the unit vector $\hat{\boldsymbol{n}}$ perpendicular to the plane shown in the figure below, i.e. the plane joining the points $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$.

4 Directional Derivative

You are on a hike. The altitude nearby is described by the function $f(x, y) = kx^2y$, where $k = 20\frac{\text{m}}{\text{km}^3}$ is a constant, x and y are east and north coordinates, respectively, with units of kilometers. You're standing at the spot $(3\text{ km}, 2\text{ km})$ and there is a cottage located at $(1\text{ km}, 2\text{ km})$. You drop your water bottle and the water spills out.

(a) Plot the function $f(x, y)$ and also its level curves in your favorite plotting software. Include images of these graphs. Special note: If you use a computer program written by someone else, you must reference that appropriately.

(b) In which direction in space does the water flow?

(c) At the spot you're standing, what is the slope of the ground in the direction of the cottage?

(d) Does your result to part (c) make sense from the graph?