1 Flux Through a Cone

(6 pts) Find the flux through a cone (with no cap!) of height H and radius R due to the vector field $\vec{F} = C\,z\,\hat{z}$. The cone has its vertex at the origin, opens upward symmetrically around the z-axis. Orient your area elements so that upward pointing fields contribute positive flux.

2 Gauss's Law on a Cylindrical Shell

(6, 2, 2 pts) Consider an infinite positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density $\rho(\vec{r}) = \alpha\, e^{(ks)^2}$.

Answer each of the following questions:

(a) Use Gauss's Law and symmetry arguments to find the electric field at each of the three radii below:

  (a) $s > b$

  (b) $a < s < b$

  (c) $s < a$

(b) What dimensions do α and k have?

(c) For $\alpha = 1$, $k = 1$, sketch the magnitude of the electric field as a function of s.

3 Spherical Shell Step Functions

(2, 2, 2 pts) One way to write volume charge densities without using piecewise functions is to use step (Θ) or delta (δ) functions.

Consider a spherical shell with charge density

$$\rho(\vec{r}) = \alpha 3 e^{(kr)^3}$$

between the inner radius a and the outer radius b. The charge density is zero everywhere else.

(a) What are the dimensions of the constants α and k?

(b) By hand, sketch a graph the charge density as a function of r for $\alpha > 0$ and $k > 0$.

(c) Use step functions to write this charge density as a single function valid everywhere in space.

4 Mass of a Slab

(2, 2, 2, 2, 2, 2 pts)

Determine the total mass of each of the slabs below.

(a) A square slab of side length L with thickness h, resting on a table top at $z = 0$, whose mass density is given by

$$\rho = A\pi \sin\left[\tfrac{\pi z}{h}\right].$$

(b) A square slab of side length L with thickness h, resting on a table top at $z = 0$, whose mass density is given by

$$\rho = 2A\Big[\Theta(z) - \Theta(z-h)\Big]$$

(c) An infinitesimally thin square sheet of side length L, resting on a table top at $z = 0$, whose surface density is given by $\sigma = 2Ah$.

(d) An infinitesimally thin square sheet of side length L, resting on a table top at $z = 0$, whose mass density is given by $\rho = 2Ah\,\delta(z)$.

(e) What are the dimensions of A?

(f) Write several sentences comparing your answers to the different cases above.