1 Current in a Wire

(4 pts) The current density in a cylindrical wire of radius R is given by $\vec{J}(\vec{r}) = \alpha s^3 \cos^2\phi\,\hat{z}$. Find the total current in the wire.

2 Divergence through a Prism

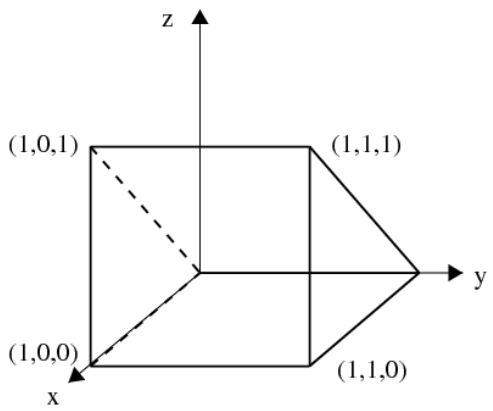
(2, 2, 4 pts)

Consider the vector field $\vec{F} = (x+2)\hat{x} + (z+2)\hat{z}$.

(a) Calculate the divergence of $\vec{F}$.

(b) In which direction does the vector field $\vec{F}$ point on the plane $z = x$? What is the value of $\vec{F}\cdot\hat{n}$ on this plane where $\hat{n}$ is the unit normal to the plane?

(c) Verify the divergence theorem for this vector field where the volume involved is drawn below. ("Verify" means calculate both sides of the divergence theorem, separately, for this example and show that they are the same.)



3 Electric Field and Charge

(4, 4, 2 pts) Consider the electric field

$$\vec{E}(r,\theta,\phi) = \begin{cases} 0 & \text{for } r < a \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{b^3-a^3}\left(r - \frac{a^3}{r^2}\right)\hat{r} & \text{for } a < r < b \\ 0 & \text{for } r > b \end{cases} \tag{1}$$

(a) Use step and/or delta functions to write this electric field as a single expression valid everywhere in space.

(b) Find a formula for the charge density that creates this electric field.

(c) Interpret your formula for the charge density, i.e. explain briefly in words where the charge is.