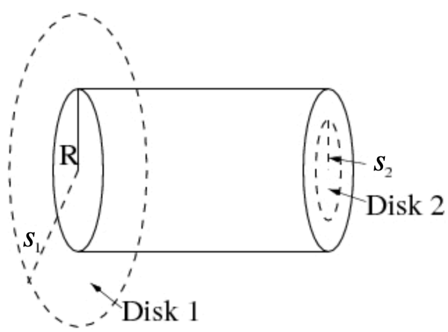
1 Ampere's Law for a Cylinder

(2, 2, 2, 2 pts)

In this problem, you will be investigating a cylindrical wire of finite radius R, carrying a non-uniform current density $J = \kappa s\, \hat{z}$, where κ is a constant and s is the distance from the axis of the cylinder.

(a) Find the total current flowing through the wire.

(b) Find the current flowing through Disk 2, a central (circular cross-section) portion of the wire out to a radius $s_2 < R$.



(c) Use Ampere's law to find the magnetic field at a distance s_1 outside the wire.

(d) Use Ampere's law to find the magnetic field at a distance s_2 inside the wire.

2 Magnetic Field and Current

(2, 4, 2 pts) Consider the magnetic field

$$\vec{B}(s, \phi, z) = \begin{cases} 0 & 0 \leq s < a \\ \alpha \frac{1}{s}(s^4 - a^4)\,\hat{\phi} & a < s < b \\ 0 & s > b \end{cases}$$

(a) Use step and/or delta functions to write this magnetic field as a single expression valid everywhere in space.

(b) Find a formula for the current density that creates this magnetic field.

(c) Interpret your formula for the current density, i.e. explain briefly in words where the current is.

3 Path Independence

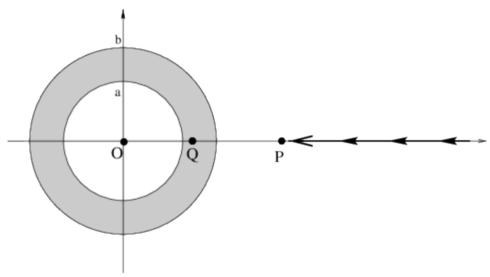
(2, 2, 2, 4, 2 pts)

The gravitational field due to a spherical shell of mass is given by:

$$\vec{g} = \begin{cases} 0 & r < a \\ -\frac{4}{3}\pi\rho G\left(r - \frac{a^3}{r^2}\right)\hat{r} & a < r < b \\ -\frac{4}{3}\pi\rho G\left(\frac{b^3 - a^3}{r^2}\right)\hat{r} & b < r \end{cases} \tag{1}$$

where a is the inside radius of the shell, b is the outside radius of the shell, and ρ is the constant mass density.

(a) Using an explicit line integral, calculate the work required to bring a test mass, of mass m_0, from infinity to a point P, which is a distance c (where $c > b$) from the center of the shell.

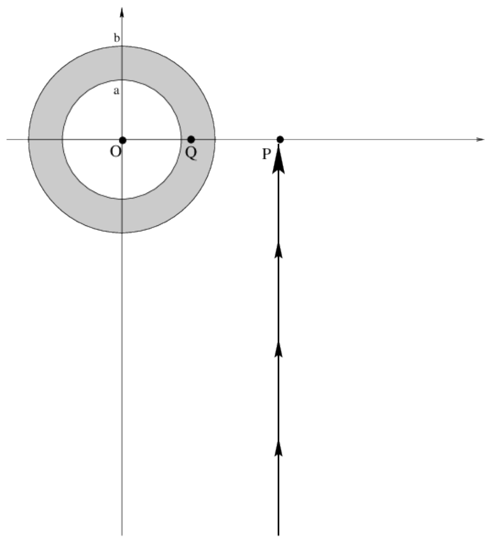


(b) Using an explicit line integral, calculate the work required to bring the test mass along the same path, from infinity to the point Q a distance d (where $a < d < b$) from the center of the shell.

(c) Using an explicit line integral, calculate the work required to bring the test mass along the same radial path from infinity all the way to the center of the shell.

(d) Using an explicit line integral, calculate the work required to bring in the test mass along the path drawn below, to the point P of the first question. Compare the work to your answer from the first question.

(e) What is the work required to bring the test mass from infinity along the path drawn below to the point P of question a. Explain your reasoning.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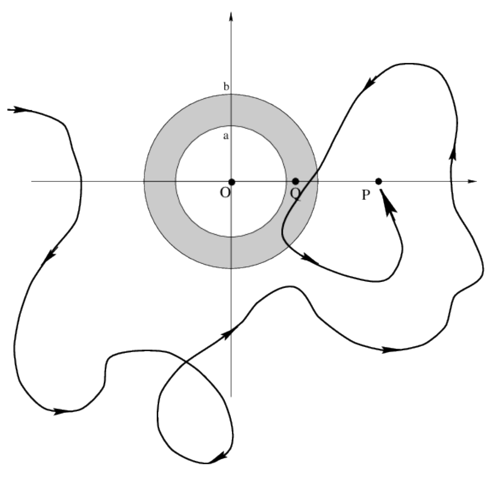
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