

Turn in this HW on or before Friday at 8pm.

## 1 Analyze TS rectangle

Consider a monatomic ideal gas that undergoes a 4-step cyclic process. On a  $T$ - $S$  diagram, the four steps of the process trace out a rectangle. The cycle proceeds in the clockwise direction around the rectangular path. The four sides of the rectangle correspond to

- (A) Constant temperature,  $T_h$
- (B) Constant entropy,  $S_h$
- (C) Constant temperature,  $T_l$
- (D) Constant entropy,  $S_l$

- (a) Make a  $T$ - $S$  diagram that represents this cyclic process. Label the axes, the four steps (A through D), the direction of each process, and the key values of  $S$  and  $T$ . The horizontal axis corresponds to which variable? Why?
- (b) Does a clockwise path in  $T$ - $S$  space correspond to a heat engine or a heat pump?
- (c) Create a table, like the one below, and fill in all the values in terms of  $T_l$ ,  $T_h$ ,  $S_l$  and  $S_h$ :

Process	$\Delta U$	$Q$	$W$
A			
B			
C			
D			

- (d) If this cycle corresponds to a heat engine, find the efficiency in terms of  $T_h$  and  $T_l$ . Alternatively, if this cycle corresponds to a heat pump, find the coefficient of performance in terms of  $T_h$  and  $T_l$ .

## 2 Helmholtz Free Energy of a Van Der Waals Gas

The Helmholtz free energy of a van der Waals (vdW) gas can be written as:

$$F = -NkT \left\{ 1 + \ln \left[ \frac{(V - Nb)T^{\frac{3}{2}}}{N} \right] \right\} - \frac{aN^2}{V}$$

Where  $a$  and  $b$  are constants.

- (a) Derive the equation of state (relationship between  $p$ ,  $T$ , and  $V$ ) for this Helmholtz free energy.  
*Hint:* The starting equations for this problem include the thermodynamic identity, the definition of Helmholtz free energy,  $F = U - TS$ , and math identities such as the overlord equation.  
**Bonus point:** Rearrange the vdW equation of state to highlight any similarities with the ideal gas equation of state ( $pV = NkT$ ). To highlight similarities, group together terms that have dimensions of pressure, group together terms that have dimension of volume, etc.

- (b) Using your expression from part (a), sketch or plot  $p(V)$  at various fixed temperatures. The volume axis should include  $Nb$  up to  $6Nb$ . Your plot can be dimensionless (i.e.  $V/Nb$  on the x axis). Select values of  $NkT$  and  $aN^2$  that give curves with different shapes. Can you create a minima in pressure near  $V = 2Nb$ ?