1 Harmonic Integrals

None
Show that:

$$\frac{2}{T}\int_0^T \sin(n\omega t)\sin(m\omega t)dt = \delta_{m,n}$$

Here the period $T = 2\pi/\omega$, and n and m are integers greater than zero. Recall that $\delta_{m,n}$ (the "Kronecker delta") is given by

$$\delta_{m,n} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

You will have to treat the two cases separately. Do not choose specific values of m and n, prove this relationship in general for ANY integer m and n.

Hints: Since it is easy to integrate exponentials, even if the exponent is a complex number, use Euler's formula to change the sines into exponentials:

$$\sin(n\omega t) = \frac{e^{in\omega t} - e^{-in\omega t}}{2i}$$

Beware of zero in the denominator of fractions!

Please evaluate all integrals analytically by hand.

2 Practice with the Kronecker Delta

None Show or complete the following: (Note: the notation $\delta_{n,m}$ is the same as δ_{nm}. I use the version with a comma when I want to write an expression for one of the integers without ambiguity or if one of the integers has two digits.)

(a)

$$\sum_{n=1}^{\infty} \delta_{n3} = 1$$

(b)

$$\sum_{n=1}^{\infty} b_n \delta_{n3} = b_3$$

(c)

$$\sum_{n=1}^{10}\sum_{m=1}^{10} \delta_{nm} = \ ?$$

(d)

$$(n-1) \times m \times \delta_{n,m+1} = \ ?$$