1 Harmonic Integrals

Show that:

$$\frac{2}{T}\int_0^T \sin(n\omega t)\sin(m\omega t)dt = \delta_{m,n}$$

Here the period $T = 2\pi/\omega$, and n and m are integers greater than zero. Recall that $\delta_{m,n}$ (the "Kronecker delta") is given by

$$\delta_{m,n} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

You will have to treat the two cases separately. Do not choose specific values of m and n, prove this relationship in general for ANY integer m and n.

Hints: Since it is easy to integrate exponentials, even if the exponent is a complex number, use Euler's formula to change the sines into exponentials:

$$\sin(n\omega t) = \frac{e^{in\omega t} - e^{-in\omega t}}{2i}$$

Beware of zero in the denominator of fractions!

Please evaluate all integrals analytically by hand.

2 Change the Period in Fourier Series

The integrals that show the Fourier series basis functions are orthonormal on the interval $x = 0 \ldots 2\pi$ are:

$$\int_0^{2\pi} \cos nx \cos mx \, dx = \pi\delta_{m,n} \qquad n, m = 1, \ldots, \infty$$

$$\int_0^{2\pi} \cos nx \cos mx \, dx = 2\pi\delta_{m,n} \qquad n = 0$$

$$\int_0^{2\pi} \sin nx \sin mx \, dx = \pi\delta_{m,n} \qquad n, m = 1, \ldots, \infty$$

$$\int_0^{2\pi} \sin nx \cos mx \, dx = 0 \qquad n, m = 1, \ldots, \infty$$

Notice that in the equations above, the variable x is dimensionless. In practical physics problems, you often want to work with a function which is periodic on the range $y = 0 \ldots L$, where y is a variable with dimensions of length. Use a simple change of variables to find equations analogous to the ones above if the interval is $y = 0 \ldots L$. Do the change of variables by hand, not with technology.