1 Frequency

(4, 2 points)

Consider a two-state quantum system (i.e., a system with a two-dimensional Hilbert space) with a Hamiltonian

$$\hat{H} \doteq \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \tag{1}$$

Another physical observable M is represented by the operator

$$\hat{M} \doteq \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} \tag{2}$$

where c is real and positive. Note: Both matrices are written in the same basis.
The initial state of the system is $|\psi(t=0)\rangle = |m_1\rangle$, where $|m_1\rangle$ is the eigenstate of $\hat{M}$ corresponding to the larger of the two eigenvalues of $\hat{M}$.

(a) What is the expectation value of M as a function of time?

(b) What is the frequency of oscillation of the expectation value of M?

2 Magnet

(2, 4, 4 points)

Consider a spin-1/2 particle with a magnetic moment. At time $t = 0$, the state of the particle is $|\psi(t=0)\rangle = |+\rangle$.

(a) If the observable S_x is measured at time $t = 0$, what are the possible measurement values, and what is the probability of obtaining each value?

(b) Instead of performing the above measurement, the system is allowed to evolve in a uniform magnetic field $\vec{B} = B_0\,\hat{y}$. The Hamiltonian for a system in a uniform magnetic field $\vec{B} = B_0\,\hat{y}$ is $\hat{H} = \omega_0\,\hat{S}_y$. (You can treat ω_0 as a given parameter in your answers to the following two questions.)

- What is the state of the system at the later time t (i.e., find $|\psi(t)\rangle$)? Express your answer as a superposition of the S_z eigenstates $|+\rangle_z$ and $|-\rangle_z$.

- At time t, the observable S_x is measured, what is the probability that a value $\hbar/2$ will be found?

3 Probabilities of Energy

(4, 4 points) (adapted from McIntyre Problem # 3.2)

(a) Show that the probability of a measurement of the energy is time independent for a general state:

$$|\psi(t)\rangle = \sum_n c_n(t) |E_n\rangle$$

that evolves due to a time-independent Hamiltonian.

(b) Show that the probabilities of measurements of other observables that commute with the Hamiltonian are also time independent (neither operator has degeneracy).