

1 Time Dependence for a Quantum Particle on a Ring

(2, 2, 2, 2 pts) Consider a quantum particle on a ring. At $t = 0$, the particle is in state:

$$|\Phi(t=0)\rangle = \sqrt{\frac{2}{3}}|-3\rangle + \frac{1}{\sqrt{6}}|-1\rangle + \frac{i}{\sqrt{6}}|3\rangle \quad (1)$$

- (a) You carry out a measurement to determine the z -component of the angular momentum of the particle at some time, $t > 0$. Calculate the probability that you measure the z -component of the angular momentum to be $3\hbar$.
- (b) You carry out a measurement to determine the energy of the particle at some time, $t > 0$. Calculate the probability that you measure the energy to be $\frac{9\hbar^2}{2I}$.
- (c) Calculate the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{3}$ at some time, $t > 0$.
- (d) Under what circumstances do measurement probabilities change with time?

2 Quantum Numbers on the Sphere

(4, 4 pts)

Consider an arbitrary state for a quantum particle confined to the surface of a sphere written as a superposition of spherical harmonics:

$$|\Psi\rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} c_{\ell m} |\ell, m\rangle \quad (2)$$

- (a) Suppose you wanted to calculate the probability that an energy measurement of this state would yield E_{17} . What coefficients $c_{\ell m}$ would you need to find to calculate this? Write an expression for this probability.
- (b) Suppose you wanted to calculate the probability that a measurement of the z -component of angular momentum would yield $5\hbar$. What coefficients $c_{\ell m}$ would you need to find to calculate this? Write an expression for this probability.

3 Sphere Questions

(2, 2, 2, 2, 2, 2 pts)

Consider the following normalized state for the rigid rotor given by:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|1, -1\rangle + \frac{1}{\sqrt{3}}|1, 0\rangle + \frac{i}{\sqrt{6}}|0, 0\rangle \quad (3)$$

- (a) Write the state as a superposition of spherical harmonics Y_l^m

- (b) What is the probability that a measurement of L_z will yield $2\hbar$? $-\hbar$? $0\hbar$?
- (c) If you measured the z-component of angular momentum to be $-\hbar$, what would the state of the particle be immediately after the measurement is made? What about if it yields $0\hbar$?
- (d) What is the expectation value of L_z in the original state $|\psi\rangle$?
- (e) What is the expectation value of L^2 in the original state $|\psi\rangle$?
- (f) What is the expectation value of the energy in the original state $|\psi\rangle$?

4 Confidence Rating

(1 pt) After solving each problem on the assignment, indicate your answers to the following questions for each problem. Answer for the problem as a whole, even if the problem has multiple parts.

- (a) **Question Confidence** How confident are you that you are interpreting the problem the way the instructor intends?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

For the rest of the questions, assume you have interpreted the problem correctly

- (b) **Problem Confidence** How confident are you that you could independently come up with a correct solution process to a similar problem on a future problem set?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

- (c) **Answer Confidence** How confident are you that your final answer to this question is correct (not solution process)?

1	2	3	4	5	6	7
Not confident at all			Somewhat confident			Extremely confident

- (d) **Makes Sense** To what degree do you understand how your answer fits (or does not fit) the physical or mathematical situation of the problem?

VN	N	LN	IDK	LF	F	VF
Very confident answer does NOT fit	Somewhat confident answer does NOT fit	Leaning toward the answer does NOT fit	Don't know if answer fits or not	Leaning toward the answer fits	Somewhat confident the answer fits	Very confident answer fits