1 Recurrence Relations

For each of the problems below, suppose you have been solving a differential equation using power series methods around the indicated point and you have derived the indicated recurrence relation. Write out the first five nonzero terms in the power series expansion. If the recurrence relation allows two solutions, write out the first four nonzero terms in each such solution.

(a) In an expansion around the point $z = 1$, the recurrence relation is:

$$a_{n+1} = \frac{1}{n+1}\, a_n$$

(b) In an expansion around the point $z = 0$, the recurrence relation is:

$$a_{n+2} = -\frac{(5-n)(6+n)}{(n+2)(n+1)}\, a_n$$

2 ODE Power Series Solutions One

Consider the differential equation $y'' - 2y' + y = 0$.

(a) Use the power series method to find the first six terms in each of two independent solutions to this differential equation.

(b) Solve this differential equation using a different method and show that your answers are the same as part a.

3 Quantum Particle in a 2D Box with Time Dependence

The eigenstates for a quantum mechanical particle inside a 2-dimensional infinite potential well with sides of length L_x, L_y are

$$|n, m\rangle \doteq \sqrt{\frac{2}{L_x}} \sin\frac{n_x \pi x}{L_x} \sqrt{\frac{2}{L_y}} \sin\frac{n_y \pi y}{L_y}$$

(a) Find an exact expression for the initial wave function given by:

$$\psi(x, y, 0) = \frac{30}{\sqrt{L_x^5 L_y^5}} (L_x x - x^2)(L_y y - y^2)$$

I have chosen coordinates so that one of the corners of the box is at the origin and all of the box is in the first quadrant (i.e. all positive values of the spatial coordinates).

(b) Find an exact expression for the wave function as a function of time.