1 Scattering

Consider a very light particle of mass μ scattering from a very heavy, stationary particle of mass M. The **force** between the two particles is a **repulsive** Coulomb force $\frac{k}{r^2}$ (neglect the gravitational force). The impact parameter b in a scattering problem is defined to be the distance which would be the closest approach if there were no interaction (See Figure). The initial velocity (far from the scattering event) of the mass μ is $\vec{v}_0$.

Answer the following questions about this situation in terms of k, M, μ, $\vec{v}_0$, and b. (It is not necessarily wise to answer these questions in order.)

(a) What is the initial angular momentum of the system?

(b) What is the initial total energy of the system?

(c) What is the distance of closest approach r_{min} **with** the interaction?

(d) Sketch the effective potential.

(e) What is the angular momentum at r_{min}?

(f) What is the total energy of the system at r_{min}?

(g) What is the radial component of the velocity at r_{min}?

(h) What is the tangential component of the velocity at r_{min}?

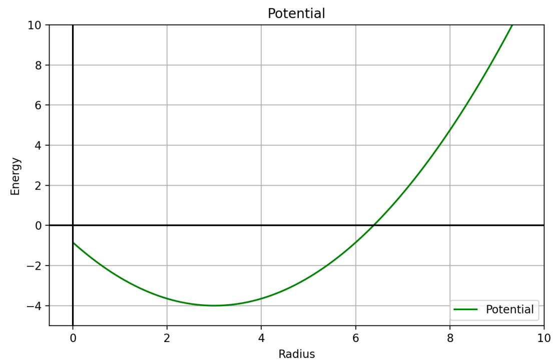
(i) What is the value of the effective potential at r_{min}?

(j) For what values of the initial total energy are there bound orbits?

(k) Using your results above, write a short essay describing this type of scattering problem, at a level appropriate to share with another Paradigm student.

2 Effective Potential Diagrams, version 2

Consider a mass μ in the potential shown in the graph below.



(a) Sketch the effective potential if the angular momentum is **not** zero.

(b) Describe qualitatively, the shapes of all possible types of orbits, indicating the energy for each in your diagram.

3 Working with Representations on the Ring

The following are 3 different representations for the **same** state on a quantum ring for $r_0 = 1$

$$|\Phi_a\rangle = i\sqrt{\frac{2}{12}}\,|3\rangle - \sqrt{\frac{1}{12}}\,|1\rangle + \sqrt{\frac{3}{12}}e^{i\frac{\pi}{4}}\,|0\rangle - i\sqrt{\frac{2}{12}}\,|{-1}\rangle + \sqrt{\frac{4}{12}}\,|{-3}\rangle \tag{1}$$

$$|\Phi_b\rangle \doteq \begin{pmatrix} \vdots \\ i\sqrt{\frac{2}{12}} \\ 0 \\ -\sqrt{\frac{1}{12}} \\ \sqrt{\frac{3}{12}}e^{i\frac{\pi}{4}} \\ -i\sqrt{\frac{2}{12}} \\ 0 \\ \sqrt{\frac{4}{12}} \\ \vdots \end{pmatrix} \leftarrow m = 0 \tag{2}$$

$$\Phi_c(\phi) \doteq \sqrt{\frac{1}{24\pi}}\left(i\sqrt{2}e^{i3\phi} - e^{i\phi} + \sqrt{3}e^{i\frac{\pi}{4}} - i\sqrt{2}e^{-i\phi} + \sqrt{4}e^{-i3\phi}\right) \tag{3}$$

(a) With each representation of the state given above, explicitly calculate the probability that $L_z = -1\hbar$. Then, calculate all other non-zero probabilities for values of L_z with a method/representation of your choice.

(b) Explain how you could be sure you calculated all of the non-zero probabilities.

(c) If you measured the z-component of angular momentum to be $3\hbar$, what would the state of the particle be immediately after the measurement is made?

(d) With each representation of the state given above, explicitly calculate the probability that $E = \frac{9}{2}\frac{\hbar^2}{I}$. Then, calculate all other non-zero probabilities for values of E with a method of your choice.

(e) If you measured the energy of the state to be $\frac{9}{2}\frac{\hbar^2}{I}$, what would the state of the particle be immediately after the measurement is made?