

## 1 Ring Function

Consider the normalized wavefunction  $\Phi(\phi)$  for a quantum mechanical particle of mass  $\mu$  constrained to move on a circle of radius  $r_0$ , given by:

$$\Phi(\phi) = \frac{N}{2 + \cos(3\phi)} \quad (1)$$

where  $N$  is the normalization constant.

- Find  $N$ .
- Plot this wave function.
- Plot the probability density.
- Find the probability that if you measured  $L_z$  you would get  $3\hbar$ .
- What is the expectation value of  $L_z$  in this state?

## 2 Time Dependence for a Quantum Particle on a Ring

Consider a quantum particle on a ring. At  $t = 0$ , the particle is in state:

$$|\Phi(t = 0)\rangle = \sqrt{\frac{2}{3}}|-3\rangle + \frac{1}{\sqrt{6}}|-1\rangle + \frac{i}{\sqrt{6}}|3\rangle \quad (2)$$

- You carry out a measurement to determine the  $z$ -component of the angular momentum of the particle at some time,  $t > 0$ . Calculate the probability that you measure the  $z$ -component of the angular momentum to be  $3\hbar$ .
- You carry out a measurement to determine the energy of the particle at some time,  $t > 0$ . Calculate the probability that you measure the energy to be  $\frac{9\hbar^2}{2I}$ .
- Calculate the probability that the particle can be found in the region  $0 < \phi < \frac{\pi}{3}$  at some time,  $t > 0$ .
- Under what circumstances do measurement probabilities change with time?

## 3 Ring Table

Attached, you will find a table showing different representations of physical quantities associated with a quantum particle confined to a ring. Fill in all of the missing entries. Hint: You may look ahead. We filled out a number of the entries throughout the table to give you hints about what the forms of the other entries might be. [pdf link for the Table](#) or [doc link for the Table](#)