1 Quantum Numbers on the Sphere

Consider an arbitrary state for a quantum particle confined to the surface of a sphere written as a superposition of spherical harmonics:

$$|\Psi\rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} c_{\ell m} |\ell, m\rangle \tag{1}$$

(a) Suppose you wanted to calculate the probability that an energy measurment of this state would yield E_{17}. What coefficients $c_{\ell m}$ would you need to find to calculate this? Write an expression for this probability.

(b) Suppose you wanted to calculate the probability that a measurement of the z-component of angular momentum would yield $5\hbar$. What coefficients $c_{\ell m}$ would you need to find to calculate this? Write an expresson for this probability.

2 Sphere Questions

Consider the following normalized state for the rigid rotor given by:

$$|\psi\rangle = \frac{1}{\sqrt{2}} |1, -1\rangle + \frac{1}{\sqrt{3}} |1, 0\rangle + \frac{i}{\sqrt{6}} |0, 0\rangle \tag{2}$$

(a) Write the state as a superposition of spherical harmonics Y_l^m

(b) What is the probability that a measurement of L_z will yield $2\hbar$? $-\hbar$? $0\hbar$?

(c) If you measured the z-component of angular momentum to be $-\hbar$, what would the state of the particle be immediately after the measurement is made? What about if it yields $0\hbar$?

(d) What is the expectation value of L_z in the original state $|\psi\rangle$?

(e) What is the expectation value of L^2 in the original state $|\psi\rangle$?

(f) What is the expectation value of the energy in the original state $|\psi\rangle$?

3 Sphere

Consider the normalized function:

$$f(\theta, \phi) = \begin{cases} N\left(\frac{\pi^2}{4} - \theta^2\right) & 0 < \theta < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < \theta < \pi \end{cases} \tag{3}$$

where

$$N = \frac{1}{\sqrt{\frac{\pi^5}{8} + 2\pi^3 - 24\pi^2 + 48\pi}} \tag{4}$$

(a) Find the coefficients of the $|\ell, m\rangle = |0, 0\rangle$, $|1, -1\rangle$, $|1, 0\rangle$, and $|1, 1\rangle$ terms in the spherical harmonic expansion of $f(\theta, \phi)$. It is helpful to remember that Mathematica has a built-in function for spherical harmonics: **SphericalHarmonicY[l, m, \[Theta], \[Phi]]**.

(b) If a quantum particle, confined to the surface of a sphere, is in the state above, what is the probability that a measurement of the square of the total angular momentum will yield $2\hbar^2$? $4\hbar^2$?

(c) If a quantum particle, confined to the surface of a sphere, is in the state above, what is the probability that the particle can be found in the region $0 < \theta < \frac{\pi}{6}$ and $0 < \phi < \frac{\pi}{6}$?