

In this course we will be taking a physics concept and applying it to a challenge. The first example doesn't require any new physics, you've learned about gravitational potential energy.

The first challenge I'll address is that of hydroelectric energy in Oregon. Do we have the potential to power the state using hydroelectric energy? We will address this by considering how much energy the Columbia river gorge *could theoretically provide*.

This kind of reasoning can relatively easily give us an upper bound on the energy available. Finding out how much we can realistically get is trickier, but often the upper bound is enough to tell us quite a bit.

We're going to be using a coarse-grained model. We don't know every detail, but are looking for a rough sketch that will give us a general idea of how much energy is available. Finally, when I crunch numbers for this, I'll be using a precision that is appropriate to the uncertainty in the model. I know there will be some assumptions and guesses, so we won't be using five decimal places.

The average flow rate of the Columbia River is  $7500 \text{ m}^3/\text{s}$  at the outlet to the Pacific Ocean. This is where most of the hydroelectric power comes from in our region. To make a coarse-grained model, we will need some observations from the real world. We're looking for what are the essential properties of the river for estimating the energy we can get from it. The flow rate is one of those things. The other thing we need is to know the height of the dams that are producing electricity. If we consider the four biggest dams along the Columbia River, they each have a vertical drop between 50 m and 60 m for a total of about 230 m.

Or alternatively we could consider the total drop in elevation. The elevation of Castlegar, BC (which is near the Columbia River where it enters the USA) is 1,476 feet, or 450 m, which puts an upper limit on the total height that could be dammed within the United States.

Simplest assumptions for building a **coarse grained model**:

- All gravitational energy is turned into electrical energy
- Flow rate of the river is the same throughout its course
- Density of water is  $1 \text{ g/cm}^3$

$$\Delta U_g = mg\Delta h \quad (1)$$

This is true for any object on the Earth as long as the height is within about 10 km of sea level.

**Challenge:** How many Joules per second can I expect from hydroelectric?

Every second  $7500 \text{ m}^3$  falls down about 230 m in total. (Or maybe as much as 450 m, if we were to build even more dams, so the river level only dropped at dams.)

$$\text{mass of water in one second} = [7500 \text{ m}^3][1000 \text{ kg/m}^3] \quad (2)$$

$$= 7.5 \times 10^6 \text{ kg} \quad (3)$$



The change in gravitational energy in one second is thus

$$\Delta U_g = mg\Delta h \quad (4)$$

$$= [7.5 \times 10^6 \text{ kg}][10 \text{ m/s}^2][230 \text{ m}] \quad (5)$$

$$= [7.5 \times 10^6 \text{ kg}][10^1 \text{ m/s}^2][2.3 \times 10^2 \text{ m}] \quad (6)$$

$$= 7.5 \times 2.3 \times 10^9 \text{ kg m}^2/\text{s}^2 \quad (7)$$

$$= 17 \times 10^9 \text{ J/s} \quad (8)$$

$$= 17 \text{ GJ/s} \quad (9)$$

Or maybe twice this if we were to build a lot more dams.

$$\text{rate of energy production} \approx 17 \text{ GJ/s} \quad (10)$$

where the  $\approx$  means “approximately equal to” which reminds us that we made some rough approximations.

When we encounter a number like 17 GJ, we need to ask how to make sense of that huge amount of energy? It depends on the question, so we need to think about context. In this case, we are thinking about energy supplied to Oregon (and maybe Washington?) which means it must be shared with 4 million people. So how much energy would each person get? We would each get

$$\text{energy per person} = \frac{17 \times 10^9 \text{ J/s}}{4 \times 10^6 \text{ people}} \quad (11)$$

$$= 4 \times 10^3 \text{ J/s per person} \quad (12)$$

$$= 4000 \text{ W per person} \quad (13)$$

This is like 5 computers running CPU-intensive tasks. Or perhaps microwave (1000 J/s) and a tea kettle (1000 J/s) and a space heater (1500 J/s). So maybe it's enough electricity to keep Oregon going, but not by much. Especially if we have to share with Washington.

**Fact check** In 2012, the Columbia river and its tributaries accounted for 29 GJ/s of electricity. This was more than our estimate based on four dams, but less than the estimate we would have achieved based on the total elevation drop in the USA.