In Basics of heat engines I showed a diagram a lot like this:

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Figure 1: Energy flow diagram for a heat engine

Energy flow diagram In this picture, we see energy flowing as heat from a hot reservoir into a mysterious machine, and then out of that machine comes both useful work and heat dumped into a cold reservoir. We are going to discuss how one might construct such a machine using a gas.

This machine could be something as simple as a balloon of air. I'm going to start with a medium-sized balloon and put it in contact with the hot reservoir where it will expand, and then move it to the cold reservoir where it will contract. We can repeat this cycle, with energy entering the balloon from the hot reservoir.

You might think that this process wouldn't do any useful work. To visualize this process, you might consider a piston instead of a balloon, so that when the gas expands it can can push against something (doing work), and when it contracts, it will presumably have work done on it. If the two are not equal, then we will be able to get net work done during a cycle, and we can use that work to propell a car or a locomotive.

0.1 Maximum efficiency

In Efficiency of a car engine we found that the efficiency of a heat engine had a maximum value set by the Second Law:

$$\frac{W}{Q_{\text{in}}} \leq 1 - \frac{T_C}{T_H} \tag{1}$$

$$\leq \text{Carnot efficiency} \tag{2}$$

This raises interesting practical questions:

1. How do I build such a machine?
    - gas cycles
    - phase change cycles
    - electron cycles ("thermopower")
2. What clever tricks and insights do we need to make an efficient machine?

0.2 How to analyze a heat engine that uses a gas cycle

Step 1: Quantify the work This requires us to compute force times distance over a single cycle. We'll be starting this below, and finishing it in Work in a gas heat engine.

Step 2: Quantify the heat This is harder, and often requires using knowledge of the internal energy to work backwards using the First Law, to find out how much energy was transfered by heating. We will do this in A gas heat engine, continued and Heat in a gas heat engine.

Step 1: Quantify the work To find the work, we will need to find the force on the piston, and the distance it moves. Let's start by imagining a pretty simple process:

t (s)	p (atm)	V (L)
0	1	0.5
10	2	0.5
20	2	1.0

A note about units For the common machines around us, the pressures are on the order of 10^5 N/m^2 = 1 atm, and the volumes are on the order of 10^{-3} m = 1 L

How much work happened?

$$W = \text{force} \times \text{distance} \tag{3}$$

We know the initial and final volume, but don't know the area of the piston A, or its depth x.

$$V_i = [0.5\text{ L}] = Ax_i \tag{4}$$
$$V_f = [1.0\text{ L}] = Ax_f \tag{5}$$
$$\text{force} = \text{pressure} \times \text{area} = pA \tag{6}$$
$$W = \text{force} \cdot \text{distance} \tag{7}$$
$$= (pA)(x_f - x_i) \tag{8}$$
$$= p(Ax_f - Ax_i) \tag{9}$$
$$= p(V_f - V_i) \tag{10}$$

So we see that we didn't need to know the area of the piston, or its depth. Thus if the pressure is constant, the work is given by

$$|W| = p|V_f - V_i| \tag{11}$$

If the volume doesn't change, then the work done is zero, regardless of whether the pressure might change.

We need a sign convention for work, and will use the following one:

$W > 0$ the energy is going into the gas, i.e. the gas is gaining energy due to working.

$W < 0$ the energy is leaving the gas, i.e. the gas is losing energy due to working.

This means that

$$W = -p\Delta V \qquad \text{if the pressure is essentially constant} \tag{12}$$

For a gas process, work is conveniently calculated using pressure and ΔV. For this reason, we frequently represent processes using pV diagrams. Here is a representation of the process we just considered:

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Figure 2: Process we just described

Frequently pV diagrams are confusing to students, because p *is not a function of* V! So these plots are more like a map of Corvallis on which you draw the route you take to campus. This relates to why we so frequently draw arrows on pV diagrams.

Examining the work during this process:

0-10 s No work is done, because the volume does not change.

10-20 s The pressure is constant, so we can use our formula

$$W = -p(V_f - V_i) \tag{13}$$
$$= -[2 \times 10^5 \text{ N/m}^2][0.5 \times 10^{-3} \text{ m}^3] \tag{14}$$
$$= -10^2 \text{ N} \cdot \text{m} \tag{15}$$
$$= -100 \text{ J} \tag{16}$$

During this process, the gas loses 100 J of energy. When gasses (or anything else) expand, the work is negative, because they can do work on their environment, e.g. by lifting up a weight.

Now, what if I reverse the process and go back to the same state that I started at?

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Figure 3: The same process, but in reverse

Using the same approach

20-30 s The pressure is constant, so we can use our formula

$$W = -p(V_f - V_i) \tag{17}$$
$$= -[2 \times 10^5 \text{ N/m}^2][-0.5 \times 10^{-3} \text{ m}^3] \tag{18}$$
$$= 10^2 \text{ N} \cdot \text{m} \tag{19}$$
$$= 100 \text{ J} \tag{20}$$

30-40 s No work is done, because the volume does not change.

In this case, the gas gains energy as it is compressed.

If we consider the combined 40 second process, we can compute the net work by adding up our two answers:

$$W_{\text{net}} = \underbrace{-100 \text{ J}}_{0\to 20 \text{ s}} + \underbrace{100 \text{ J}}_{20\to 40 \text{ s}} = 0 \tag{21}$$

Looking at this result, you might be tempted to conclude that no work was done because we returned to the same state state we started from. **That is not true!** As you will see momentarily in Work in a gas heat engine, the net work is not always zero, even when you return to the same state that you started at. The net work in this case is zero *because we returned by the same path.*