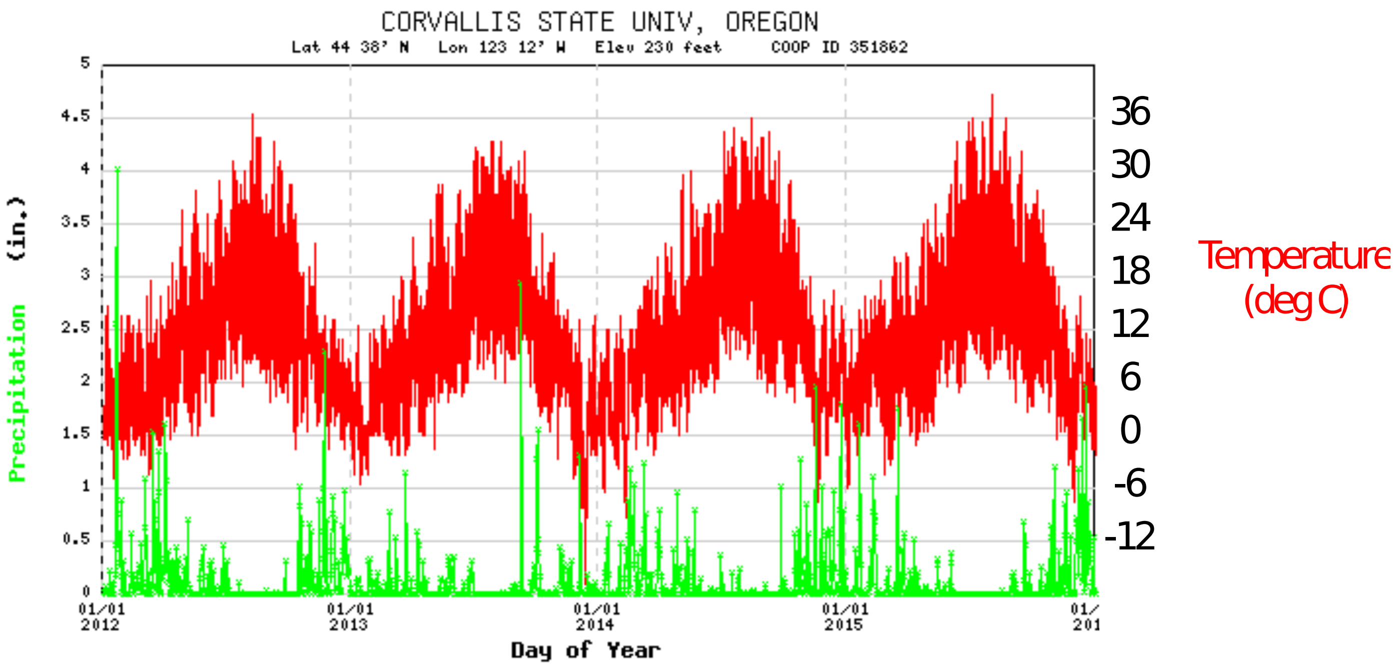

Figure 1: Daily minimum and maximum temperature (red) and daily precipitation (green) in Corvallis from 2012 to 2016.

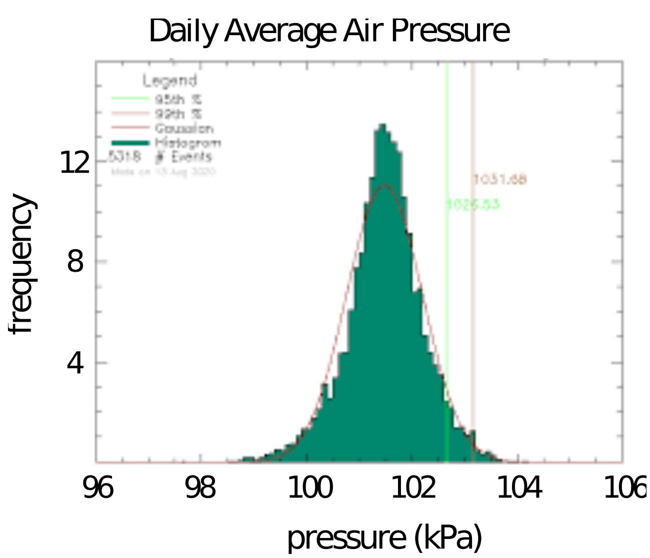

Figure 2: Distribution of atmospheric air pressure.

Assume that the random ups/downs of temperature and air pressure are independent. If I measured **air density** (kg/m^3) at random times of year and random times of day, what would be the standard deviation of my measurements?

To answer this question, estimate the standard deviation of T and p from the graphs, then calculate the standard deviation of air density as a percentage of the average air density.

You will need the relationship $pV = Nk_BT$.

Solution

- Output $f \leftrightarrow \rho$ air density
- Parameter $a \leftrightarrow p$ pressure
- Parameter $b \leftrightarrow T$ temperature

Our model is based on the ideal gas law.

$$\rho = (\text{mass of an air molecule}) \cdot \frac{N}{V} \tag{1}$$

$$= \frac{(\text{mass of an air molecule})}{k_B} \cdot \frac{p}{V} \tag{2}$$

$$= (\text{constant}) \cdot \frac{p}{V} \tag{3}$$

Now we need to estimate the uncerainty in p and T from the provided graphs.

$$\frac{\sigma_p}{p} = \frac{1\text{ kPa}}{101.5\text{ kPa}} \approx 1\% \qquad \frac{\sigma_T}{T} = \frac{14\text{ K}}{290\text{ K}} \approx 4.8\% \tag{4}$$

(Note: In practice, I would write 5% instead of 4.8%, but I kept the extra decimal place so we could actually observe the effect of the 1% on the total.)

The percentage uncertainty in air density is then

$$\frac{\sigma_\rho}{\rho} = \sqrt{(1\%)^2 + (4.8\%)^2} = 4.9\% \tag{5}$$

We see that we could have gotten the same answer by just considering the temperature fluctuations. That is yet another simplified method of combining uncertainties, which works when one of the uncertainties dominates like this.

Bonus uncertainty Suppose you attempted to measure the mean temperature in Corvallis by taking 4 measurements of the temperature at randomly chosen dates and times and computed the average of those four measurements. What would the uncertainty of the mean be?

Solution Our model is

$$\langle T \rangle = \frac{T_1 + T_2 + T_3 + T_4}{4} \tag{6}$$

We can use the same formula (but for addition) repeatedly. First we find the uncertainty of the sum of two temperature measurements

$$\sigma_{T+T} = \sqrt{\sigma_T^2 + \sigma_T^2} \tag{7}$$
$$= \sqrt{2}\sigma_T \tag{8}$$

Now we add two such sums together

$$\sigma_{T+T+T+T} = \sqrt{\sigma_{T+T}^2 + \sigma_{T+T}^2} \tag{9}$$
$$= \sqrt{2}\sigma_{T+T} \tag{10}$$
$$= 2\sigma_T \tag{11}$$

Now that we have computed the uncertainty of the sum of the four measurements, we need to compute the uncertainty of the average by dividing by four. The uncertainty in the number 4 is zero, so you could use the product uncertainty formula to find that the fractional uncertainty of $\frac{T_1+T_2+T_3+T_4}{4}$ is just the same as the fractional uncertainty of $T_1 + T_2 + T_3 + T_4$. Thus

$$\frac{\sigma_{\langle T\rangle}}{\langle T\rangle} = \frac{\sigma_{T+T+T+T}}{T_1 + T_2 + T_3 + T_4} \tag{12}$$
$$= \frac{2\sigma_T}{4T} \tag{13}$$
$$= \frac{1}{2}\frac{\sigma_T}{T} \tag{14}$$
$$\approx 2.4\% \tag{15}$$

So by taking four times as many measurements we can cut our uncertainty by $\frac{1}{2}$. This $1/\sqrt{N}$ behavior is a universal property of both uncorrelated measurements and Monte Carlo simulations.