Last time (in Travelling wave solution) you worked out an expression for a particular travelling wave. Today we're going to look a little more into what these solutions might look like.

The function you came up with was

$$y(x,t) = [0.1\text{ m}] \cos\left(\frac{2\pi}{[8m]}(x - [0.5\text{ m/s}]t)\right) \tag{1}$$

FIXME add figure of parabola offset to the right

Figure 1: A useful concept: you can subtract a number from the x coordinate of a function to make it slide to the right. Here I shift x^2 by 1 m by plotting $(x - 1\text{ m})^2$.

We give names to the parameters that specify a traveling sinusoidal wave.

$$y(x,t) = \underbrace{A}_{\text{amplitute}} \cos(\underbrace{k}_{\text{wave number}}(x - \underbrace{v}_{\text{phase velocity}} t)) \tag{2}$$

What waves are allowed on a slack line?

$$\frac{\partial^2 y}{\partial t^2} = \frac{TL}{m}\frac{\partial^2 y}{\partial x^2} \tag{3}$$

We can start by taking the derivatives we will need.

$$\frac{\partial y}{\partial x} = -A\sin\left(k(x - vt)\right)k \tag{4}$$

$$\frac{\partial^2 y}{\partial x^2} = -A\cos\left(k(x - vt)\right)k^2 \tag{5}$$

$$= -k^2 y \tag{6}$$

$$\frac{\partial y}{\partial t} = -A\sin\left(k(x - vt)\right)(-kv) \tag{7}$$

$$\frac{\partial^2 y}{\partial t^2} = -A\cos\left(k(x - vt)\right)k^2 v^2 \tag{8}$$

$$= -k^2 v^2 y \tag{9}$$

And now we can just plug in y from (2) into (3).

$$\cancel{-k^2} v^2 y = \frac{TL}{m}(\cancel{-k^2} y) \tag{10}$$

$$v^2 = \frac{TL}{m} \tag{11}$$

$$v = \pm\sqrt{\frac{TL}{m}} \tag{12}$$

so we find that we can't have any traveling wave solution we want, but just ones that have precisely the right speed.

Note 1 What I just did was solving a differential equation. Some math classes make it sound mysterious or hard. But the only way to solve a differential equation is to guess the form of the answer (we guessed a traveling wave) and then show that is correct (and find out which parameters make it correct) by taking derivatives and doing some algebra. Not that you won't learn anything useful from your math class (in particular, the concept of homogeneous and inhomogeneous solutions is incredibly powerful), but students sometimes miss out on the fundamental of what it means to solve a differential equation.

Note 2 The above proof that we found a solution in no way hinges on having chosen a sinusoidal traveling wave. We could instead have chosen any other function of $x - vt$ and we would have found that it was a solution. Most differential equations have *many* different solutions.