Estimate the optical depth for infrared light with wavelength 15 μm when it travels through our atmosphere at standard temperature and pressure STP. The number density of air molecules at STP can be found from $pV = Nk_BT$.

Our air is 0.041% CO_2 (410 ppm).

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Figure 1: Absorption cross sections σ_a. The graph is from *Fundamentals of Atmospheric Radiation* by Craig Bohren, a highly recommended book.

Solution We need to calculate the optical depth $= \frac{1}{n\sigma_a}$. From the plot

$$\sigma_a = 10^{-11}\ \mu\text{m}^2 = 10^{-23}\ \text{m}^2 \tag{1}$$

n is the number of CO_2 molecules per unit volume.

We know that there are 410 CO_2 molecules for every 10^6 air moleculte s (410 ppm). This is equivalent to 0.041%.

How many air molecules in 1 m^3?

$$pV = Nk_BT \tag{2}$$

$$\frac{N}{V} = \frac{p}{k_BT} = \frac{10^5\ \text{N/m}^2}{[1.4 \times 10^{-23}\ \text{J/K}][300\ \text{K}]} \tag{3}$$

$$= \frac{1}{4.2} \times 10^{26}\ \text{m}^{-3} \tag{4}$$

$$= 2.4 \times 10^{25}\ \text{m}^{-3} \tag{5}$$

The next step is to find the number of CO_2 molecules per m^3 from the fraction of 2.4×10^{25}.

$$N_{CO_2} = [4.1 \times 10^{-4}][2.4 \times 10^{25}]\ \text{m}^{-3} \tag{6}$$

$$= 10^{22}\ \text{m}^{-3} \tag{7}$$

Therefore the optical depth is

$$L_{\text{opt}} = \frac{1}{n\sigma} \approx \frac{1}{[10^{22}\ \text{m}^{-3}][10^{-23}\ \text{m}^2]} \tag{8}$$

$$= 10\ \text{m} \tag{9}$$

Sensemaking 10 m is much less than the thickness of the atmosphere (the atmosphere is 10 km). The light with wavelength 15 μm will not go straight up into outer space. The light will bounce around, distributing its energy into the internal vibrations of air molecules.