One of the pathways for hydrogen fusion involves the following reaction:

$$^{2}\mathrm{H} +^{3}\mathrm{H} \rightarrow^{4}\mathrm{He} + +^{1}_{0}\mathrm{n} \tag{1}$$

1. How much kinetic energy does a deuterium nucleus need to fuse with a tritium nucleus? *Note: for a course-grained approximation, you may assume that the tritium is fixed in position and the deuterium speeds toward it.*

Solution We first need to find out how big the nuclei are using

$$r_{\mathrm{nucleus}} \approx [1.2\ \mathrm{fm}]A^{\frac{1}{3}} \tag{2}$$

$$= [1.2\ \mathrm{fm}]2^{\frac{1}{3}} \text{ and } [1.2\ \mathrm{fm}]3^{\frac{1}{3}}\frac{1}{3} \tag{3}$$

$$= 1.5\ \mathrm{fm} \text{ and } 1.7\ \mathrm{fm} \tag{4}$$

Taken together, we'll say that the nuclei need to get within 3 fm to fuse.
We use the Coulomb energy:

$$U_{\mathrm{elec}} = k_C\frac{q_1 q_2}{r} \tag{5}$$

$$= \left[9\times 10^{9}\frac{\mathrm{J}\cdot\mathrm{m}}{\mathrm{C}^2}\right]\frac{[1.6\times 10^{-19}\ \mathrm{C}]^2}{3\times 10^{-15}\ \mathrm{m}} \tag{6}$$

$$\approx 8\times 10^{-14}\ \mathrm{J} \tag{7}$$

2. If the deuterium is in a gas, what temperature should the gas be so that most deuterium has enough kinetic energy for fusion?

Solution Equipartition theorem tells us that the kinetic energy of each atom is $\frac{3}{2}k_BT$. Since we have two atoms involved, we can have twice that energy available for a collision $(3k_BT)$ that. So we can find the temperature needed by setting this equal to the energy needed to touch the two nuclei

$$T = \frac{U_{\mathrm{elec}}}{3k_B} \tag{8}$$

$$= \frac{8\times 10^{-14}\ \mathrm{J}}{3\cdot 1.4\times 10^{-23}\ \mathrm{J/K}} \tag{9}$$

$$\approx 2\times 10^{9}\ \mathrm{K} \tag{10}$$

This is rather hot, far hotter than the surface of the sun. It's also why we don't yet have practical fusion reactors.

3. **Extra** Based on the masses of deuterium (^{2}H) and tritium (^{3}H), and given that one neutron will be produced, how much energy will be generated when they fuse?

isotope	mass
^{2}H	2.0141 u
^{3}H	3.01605 u
^{4}He	4.0260 u
$^{1}_{0}$n	1.0087 u

Solution The change in mass is

$$\begin{aligned} \Delta m &= 2.0141\text{ u} + 3.01605\text{ u} - 4.0260\text{ u} - 1.0087\text{ u} && (11)\\ &= -0.00455\text{ u} && (12)\\ &= -7.6\times 10^{-30}\text{ kg} && (13)\end{aligned}$$

It's good news that the mass decreased, since otherwise we wouldn't get any energy out of the reaction.

$$\begin{aligned} \text{energy generated} &= |\Delta m|c^2 && (14)\\ &= [7.6\times 10^{-30}\text{ kg}][3\times 10^{8}\text{ m/s}]^2 && (15)\\ &= 7\times 10^{-13}\text{ J} && (16)\end{aligned}$$

This is a lot less energy than we got out of splitting one uranium atom.

4. **Extra** What mass of deuterium + tritium would need to be fused to provide the electrical power the United States uses in a day?[1]

Solution

$$\begin{aligned} \text{mass needed per day} &= \frac{[10^{12}\text{ J/s}]}{[7\times 10^{-13}\text{ J/pair of atoms}]}[8\times 10^{-27}\text{ kg/pair of atoms}][9\times 10^{4}\text{ s/day}] && (17)\\ &\approx 10^{3}\text{ kg/day} && (18)\end{aligned}$$

So we do need a lot more tritium and deuterium per day than we do uranium to power the United States. But it's still 4×10^{6} times less mass than the amount of coal we need per day. Plus it wouldn't produce any carbon dioxide, which is a major plus.

[1] The electrical power use of the US is about 10^{12} J/s.