



In rectangular coordinates, the natural unit vectors are $\{\hat{x}, \hat{y}\}$, which point in the direction of increasing x and y , respectively. Similarly, in polar coordinates the natural unit vectors are \hat{r} , which points in the direction of increasing r , and $\hat{\phi}$, which points in the direction of increasing ϕ .

The *unit tangent vector* to a parametric curve is the unit vector tangent to the curve which points in the direction of increasing parameter. The *principal unit normal vector* to a parametric curve is the unit vector perpendicular to the curve “in the direction of bending”, which is the direction of the *derivative* of the unit tangent vector.

1. Consider the parametric curve $\vec{r} = 3 \cos \phi \hat{x} + 3 \sin \phi \hat{y}$ with $\phi \in [0, 2\pi]$. Calculate the unit tangent vector \hat{T} and the principal unit normal vector \hat{N} for this curve in terms of \hat{x} and \hat{y} .
2. Consider a circle of radius 3 centered at the origin. Determine the unit tangent vector \hat{T} and the principal unit normal vector \hat{N} for this curve in terms of \hat{r} and $\hat{\phi}$.
3. Compare your answers.