

In this course I expect you to learn to communicate effectively using math. Effective mathematical communication enables others to read and evaluate (and learn from!) your work. This in turn allows you to work with others to solve problems collaboratively.

Mathematical communication skills also serve as an effective mechanism for external cognition. Our brains (while powerful) are very limited machines. To solve challenging physics problems we need ways to offload our ideas and store them for later use.¹ Learning to organize your reasoning more clearly on paper will enable you to more easily and accurately solve more challenging problems. You can read and evaluate your own work, and can go back and check your work, and to find errors that you have made. If you're interested in reading more about these ideas, this New York Times article gives a layperson's introduction to external cognition.

I'll note that calculators and computers also function as a form of external cognition, as well as a means of communication.

Equations All mathematical expressions must be part of an equation (or an inequality). An isolated mathematical expression like

$$\frac{5 \times 10^3 \text{ J}}{5 \times 10^7 \text{ J/s}} \quad (1)$$

does not convey any meaning. It is neither true nor false, and is analogous to a sentence with no verb.

Starting equations A **starting equation** is an equation that does not arise from equations you already have on your sheet. Thus you may have several starting equations in a given problem. Most often your starting equations will be physical laws, although in some cases your starting equations may simply be conversions. When the source of a starting equation is not obvious, you must describe where it came from (or why it is true) in words.

Following equations A **following equation** is an equation that can be mathematically shown to be true based on equations already present. In many cases, following equations do not require words to explain them, however if it is not obvious what you did (particularly on long problems) then words will be important.

Equation sequences One common idiom in mathematical communication is an **equation sequence**, in which one side of the equation does not change:

$$\vec{F} = m\vec{a} \quad (2)$$

$$= m \frac{d^2 \vec{r}}{dt^2} \quad (3)$$

$$= -mx_0\omega^2 \sin \omega t \quad (4)$$

In this idiom, we do not rewrite the left-hand side of the equation, but it is taken to be identical. Each expression on the right-hand side in this example is equal to the force.

¹Those of you who are programmers may help to think of paper as RAM and cache, while our brains only contain registers.

Defining variables Any variables you use must be defined. This can happen earlier in the problem, or in a figure, or right after an equation with a phrase like “where m is the mass of the particle.”

0.1 Serious errors in mathematical writing

There are a few aspects of writing up a solution that are absolutely required. It is your job to communicate your solution to the grader, not the grader’s job to figure out what you were thinking (and this applies to exams as well as homework). Here are some ways that you can get points docked:

Undefined variable Use a variable without saying what it means. An exception would be if the variable is commonly used to indicate the quantity of interest and is unambiguous, e.g. using m_{electron} for the mass of an electron, or even just m for mass if there is only a single mass in the problem.

Undefined equation Introduce an equation involving numerical quantities without defining what that combination of quantities relates to.

$$\frac{5 \times 10^7 \text{ J}}{5 \times 10^3 \text{ J/s}} = 10^4 \text{ s} \quad (5)$$

This equation is true, but it doesn’t tell us what (if any) physical quantity happens to have a value of ten thousand seconds. It may be obvious to you what you’re doing, but your job is to make it obvious to the reader.

Undefined quantity Introduce numerical values without defining them.

$$\text{time to charge the battery} = \frac{5 \times 10^7 \text{ J}}{5 \times 10^3 \text{ J/s}} = 10^4 \text{ s} \quad (6)$$

This equation is problematic if I can’t tell what the value $5 \times 10^7 \text{ J}$ is. A better answer would say something like:

$$\text{time to charge the battery} = \frac{\text{storage capacity of battery}}{\text{power of charger}} = \frac{5 \times 10^7 \text{ J}}{5 \times 10^3 \text{ J/s}} = 10^4 \text{ s} \quad (7)$$

Missing units When you write out math involving numbers, you must always write the units down for any quantity that is not dimensionless.

Inconsistent dimensions An error in which two things are added, subtracted, or stated to be equal, when the two things have different dimensions.

These same considerations apply to exam work as well.

1 Homework rubric

A question is worth 3 points. You gain points by demonstrating that you know how to solve a problem. You lose points by failing to communicate effectively. Note that being able to obtain a correct numerical answer *is important!*

Clearly spent enough time to finish problem	+1	physics
Demonstrated grasp of basic principles	+1	physics
Got the correct answer	+1	physics
Units or dimensions wrong	−1 for each instance	physics
Undefined variable	−1 for each instance	communication
Undefined equation	−1 for each instance	communication
Undefined quantity	−1 for each instance	communication