To create a first-order refinement of the zero-th order model at the beginning Temperature of the Earth, we will model the atmosphere as a plexiglass sphere around the earth.

zeroth-order model first-order model

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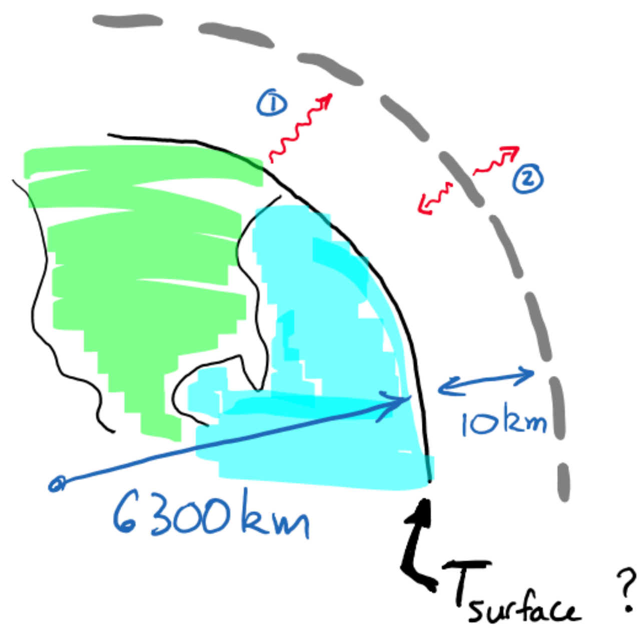
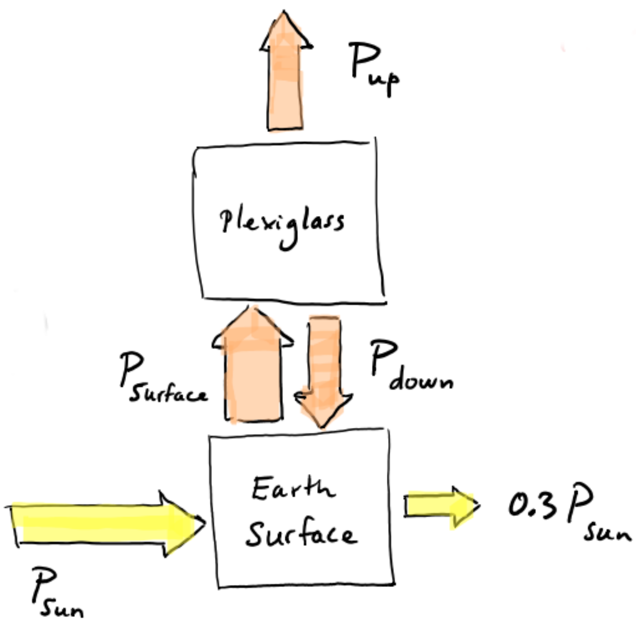


Now, what will be the surface temperature of the Earth? We will assume that the plexiglass is transparent to the light from the sun (which is largely visible and near infrared), but absorbs the light coming from the earth, which is in the far infrared region of the spectrum.

First, we'll construct an energy flow diagram:



Since the system is in steady state, the Earth and the plexiglass must not be gaining or losing energy. That tells us that

$$P_{\text{surface}} + 0.3P_{\text{sun}} = P_{\text{sun}} + P_{\text{down}} \quad \text{Earth at constant energy} \tag{1}$$

$$P_{\text{up}} + P_{\text{down}} = P_{\text{surface}} \quad \text{Plexiglass at constant energy} \tag{2}$$

We can either do a little algebra on the above, or we could think about the combined Earth-plexglass system having constant energy to find that

$$P_{\text{up}} + 0.3P_{\text{sun}} = P_{\text{sun}} \quad \text{Earth + plexiglass at constant energy} \tag{3}$$

which tells us that since the plexiglass is glowing as a blackbody with surface area equal to that of the Earth.

$$
\begin{aligned}
P_{\text{up}} &= 0.7 P_{\text{sun}} = 4\pi R_{\text{earth}}^2 \sigma T_{\text{plexi}}^4 && (4)\\
0.7\left[1360\frac{\text{J}}{\text{s}\cdot\text{m}^2}\right]\cancel{\pi R_{\text{earth}}^2} &= 4\cancel{\pi R_{\text{earth}}^2}\sigma T_{\text{plexi}}^4 && (5)\\
T_{\text{plexi}} &= 255\text{ K} && (6)
\end{aligned}
$$

Note that our math for the plexiglass temperature was *actually identical* to the zero-order model in Temperature of the Earth. But now we are interpreting that math differently now, since this is the temperature of the plexiglass *not* of the Earth's surface.

Now we can consider the equation for just the plexiglass. Here we can recognize that $P_{\text{up}} = P_{\text{down}}$, since the plexiglass will emit the same on each side, assuming the sides are at the same temperature.

$$
\begin{aligned}
\cancel{\sigma} T_{\text{surface}}^4 \cancel{4\pi R_{\text{earth}}^2} &= \cancel{\sigma} T_{\text{plexi}}^4 \cancel{4\pi R_{\text{earth}}^2} + \cancel{\sigma} T_{\text{plexi}}^4 \cancel{4\pi R_{\text{earth}}^2} && (7)\\
T_{\text{surface}}^4 &= 2T_{\text{plexi}}^4 && (8)\\
&= (1.19)255\text{ K} && (9)\\
&= 303\text{ K} && (10)
\end{aligned}
$$

This is a lot closer to the actual average temperature of 287K, but is still not quite there.

What are we omitting, and how could we improve this?

0.1 Computational climate modeling

Real computer modeling of the climate includes:

- Many layers, and a gradient of temperatures in the atmosphere.
- Some wavelengths of IR (infrared) can pass straight through.
- Tell us the upward IR flus leaving the top layer of the atmosphere, as a function of T_{surface}.

An example of a computer model: `https://paradigms.oregonstate.eduhttp://climatemodels.uchicago.edu/modtran/`

You can run this (very simple) model in your browser, and play with the concentration of CO_2 and CH_4.

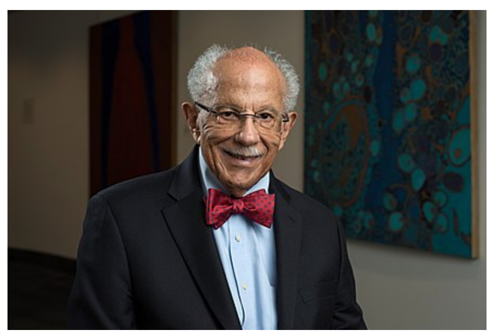


Figure 1: One of the pioneers of computational climate modeling was Warren Washington (B.S. in Physics from Oregon State University in 1958). He was a science advisor to Ronald Reagan, Jimmy Carter, George W. Bush, Bill Clinton, George H.W. Bush, and Barak Obama. President Obama presented him with the National Medal of Science in 2010.

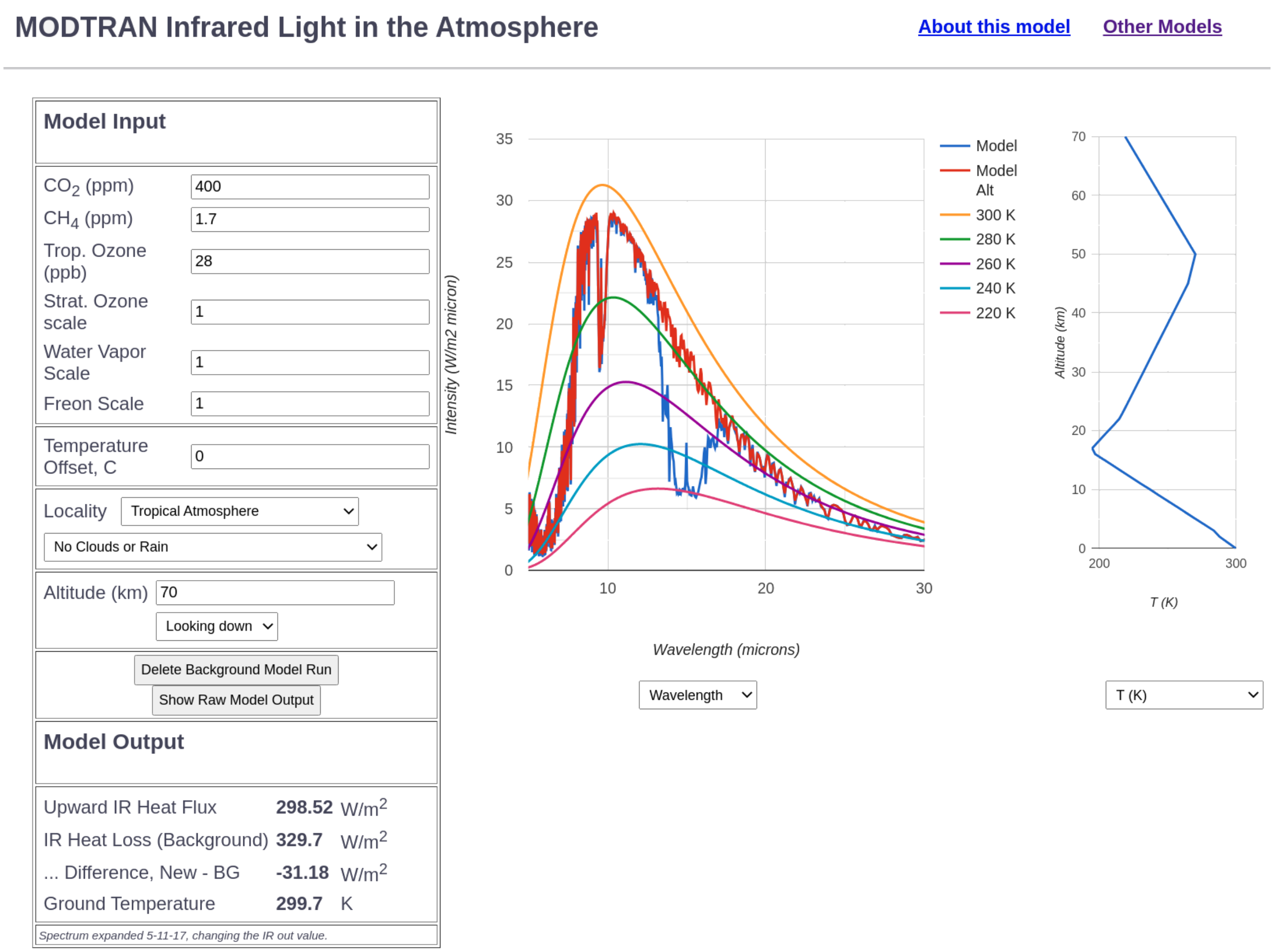


Figure 2: Screenshot from `https://paradigms.oregonstate.eduhttp://climatemodels.uchicago.edu/modtran/`. The red curve is the simulation if we set the CO_2 concentration to 0 ppm, and the blue curve is with the current CO_2 concentration of 400 ppm. Before the industrial revolution, the CO_2 concentration was 280 ppm.

Results from computer models are summarized by **radiative forcing**. Radiative forcing is difference between the intensity of sunlight absorbed by the Earth and the energy radiated back into space.

Current greenhouse gas concentrations[55]

Gas	Pre-1750 tropospheric concentration[56]	Recent tropospheric concentration[57]	Absolute increase since 1750	Percentage increase since 1750	Increased radiative forcing (W/m^2)[58]
Carbon dioxide (CO_2)	280 ppm[59]	411 ppm[60]	131 ppm	47 %	2.05[61]
Methane (CH_4)	700 ppb[62]	1893 ppb /[63][64] 1762 ppb[63]	1193 ppb / 1062 ppb	170.4% / 151.7%	0.49
Nitrous oxide (N_2O)	270 ppb[58][65]	326 ppb /[63] 324 ppb[63]	56 ppb / 54 ppb	20.7% / 20.0%	0.17
Tropospheric ozone (O_3)	237 ppb[56]	337 ppb[56]	100 ppb	42%	0.4[66]

Figure 3: Data from `https://paradigms.oregonstate.eduhttps://en.wikipedia.org/wiki/Greenhouse_gas`. In addition, halocarbons have added another 0.36 W/m^2 of forcing (listed individually in a separate table), for a total of 3.5 W/m^2. Data taken from Wikipedia 2021, when the total radiative forcing had increased by 0.5 W/m^2 over when this course was first developed.

We have an energy flow diagram in which the incoming sunlight has an energy that is $\left[3.5\frac{\text{W}}{\text{m}^2}\right] 4\pi R_{\text{Earth}}^2$ greater than the energy the Earth radiates outwards. We then want to know how this will affect the T_{surface} must increase in order to reach a new steady state equilibrium?

Quick calculation Human activity changed the atmosphere. Upward IR flus dropped from around 240 W/m^2 by about 3.5 W/m^2 (or 1.5%). How much must the temperature change in order to restore the upward energy flux to its former value? Roughly speaking, we expect the upward flux to be

$$\text{upward energy flux} = (\text{constant})\sigma T_{\text{surface}}^4 \tag{11}$$

where the constant describes the effect of the atmosphere. It's not really a constant (i.e. it has temperature dependence), but it's not unreasonable to approximate it as a constant, since the temperature won't change by a large fraction. Greenhouse gas emissions have reduced this "constant" by about 1.5%. The key factor in this equation is that the temperature is taken to the 4th power. To return to steady state, the temperature must increase by a factor $1.015^{\frac{1}{4}} = 1.0037$ or 0.37%. Thus we predict that the mean surface temperature will increase from 287 K to 288.1 K, or by 1.1 K (or 2 degrees Fahrenheit).

This result is consistent with the IPCC Report 2018, which states that

> Human activities are estimated to have caused approximately 1.0°C of global warming above pre-industrial levels, with a *likely* range of 0.8°C to 1.2°C. Global warming is *likely* to reach 1.5°C between 2030 and 2052 if it continues to increasee at the current rate. *(high confidence)*

Feedback mechanisms (Now the computer models get more complicated)

Positive feedback:

- Melting ice releases CO_2 and CH_4 bubbbles trapped in the ice.
- Less snow and ice means less sunlight is reflected and the ground absorbs more energy from the sun.
- With hotter weather, humans use more A/C.
- If the weather kills plants, it can disrupt the carbon cycle.
- Water vapor (also a greenhouse gas) increases with T.
- Clouds keep the Earth glow trapped.

Negative feedback:

- Trees grow faster with more CO_2.
- Clouds reflect more sunlight.

Today's computer models account for

- Natural feedback mechanisms.
- Local differences in reflectivity and absorption.
- Local differences in surface temperature.
- etc.