

Consider the Lie algebra $\mathfrak{so}(5, 1)$ corresponding to the orthogonal group $SO(5, 1)$.

1. What size are the matrices $X \in \mathfrak{so}(5, 1)$?
2. What is the dimension of the vector space $\mathfrak{so}(5, 1)$? That is, how many independent rotations $M(\alpha) \in SO(5, 1)$ are there? Equivalently, how many independent generators $M'(0) \in \mathfrak{so}(5, 1)$ are there?
3. The Killing form B is an inner product on $\mathfrak{so}(5, 1)$ (although not necessarily positive definite). What size is the matrix representation of B ?
4. What is the signature of B ? That is, how many rotations are there, and how many boosts?